European Journal of Operational Research 000 (2016) 1–10

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Ranking robustness and its application to evacuation planning[☆]

Marc Goerigk[a], Horst W. Hamacher[b,*], Anika Kinscherff[b]

[a] Department of Management Science, Lancaster University, Lancaster LA1 4YX, United Kingdom
[b] Fachbereich Mathematik, University of Kaiserlautern, 67663 Kaisерslautern, Germany

ARTICLE INFO

Article history:
Received 22 January 2016
Accepted 21 May 2016
Available online xxx

Keywords:
Robust optimization
Solution ranking
Combinatorial optimization
Evacuation planning

ABSTRACT

We present a new approach to handle uncertain combinatorial optimization problems that uses solution ranking procedures to determine the degree of robustness of a solution. Unlike classic concepts for robust optimization, our approach is not purely based on absolute quantitative performance, but also includes qualitative aspects that are of major importance for the decision maker.

We discuss the two variants, solution ranking and objective ranking robustness, in more detail, presenting problem complexities and solution approaches. Using an uncertain shortest path problem as a computational example, the potential of our approach is demonstrated in the context of evacuation planning due to river flooding.

1. Introduction

In the recent past, several evacuations became necessary due to river flooding in various parts of the world. From a planning point of view, operations research methods have a high potential to be used quite successfully in this context (see, for instance, Hamacher & Tjandra, 2001), since there is usually some time before the decision for an evacuation is made and the actual evacuation is started. Obviously, the water level in flooded areas is dependent on the rain fall causing the flooding, and the latter is subject to uncertainty. Therefore, robust optimization models are very appropriate to deal with flood evacuation.

Since its first formalization in the late 90s, robust optimization has seen uninterrupted rising interest both from the research community as well as from practitioners. Following the seminal work (Ben-Tal & Nemirovski, 1998), many different variants have evolved, each catering to the specialized needs of some application, or a better trade-off between conservatism and costs. We refer to Bertsimas, Brown, and Caramanis (2011), Ben-Tal, Ghaoui, and Nemirovski (2009), Goerigk and Schöbel (2016) for surveys on the topic, and to Gorissen, Yanıkoğlu, and den Hertog (2015), Chassein and Goerigk (2016b) for more hands-on guides on robust optimization.

In this paper we focus on combinatorial optimization problems with uncertain cost coefficients. As a typical example, consider a shortest path problem in a road network, where the time to traverse an edge is not known exactly, and even no probability distribution is available. More formally, we write

$$P(c) \quad \min\{f(x,c)\ :\ x\in\mathcal{X}\},\quad c\in\mathcal{U} \tag{1}$$

where $\mathcal{X}$ denotes the set of feasible solutions, and $\mathcal{U}$ a set of possible scenarios, the so-called uncertainty set.

As noted above, there exist many approaches to reformulate this family of problems $P(c)$ to a robust counterpart, whose optimal solution should perform "well" over all possible scenarios in some sense that needs to be specified. For this type of problems, we refer to the overview (Aissi, Bazgan, & Vanderpooten, 2009).

In this paper, we restrict ourselves to two classical robust counterparts. The first one, *minmax robustness* (also known as strict robustness)

$$MM \quad \min\left\{\max_{c\in\mathcal{U}} f(x,c)\ :\ x\in\mathcal{X}\right\} \tag{2}$$

is a conservative measure based on the absolute objective values of all scenarios. The second one uses a relative measure comparing objective values of a given solution with the best possible one and

☆ Partially supported by the German Ministry of Research and Technology (BMBF), projects RobEZiS, Grant number 13N13198 and StanLay, Grant number 13N12826, and by the Air Force Office of Scientific Research, Air Force Material Command, USAF, Grant number FA8655-13-1-3066. The U.S Government is authorized to reproduce and distribute reprints for Governmental purpose notwithstanding any copyright notation thereon.

* Corresponding author. Tel.: +49 6312052267.
E-mail addresses: m.goerigk@lancaster.ac.uk (M. Goerigk), hamacher@mathematik.uni-kl.de (H.W. Hamacher), kinscherff@mathematik.uni-kl.de (A. Kinscherff).

http://dx.doi.org/10.1016/j.ejor.2016.05.037

Table 1
Objective values of an example problem.

	A	*B*	*C*
c_1	50	21	10
c_2	100	105	110

is known as *minmax regret*:

$$MMR \quad \min\left\{\max_{c\in\mathcal{U}}(f(x,c) - f^*(c)) \;:\; x\in\mathcal{X}\right\}. \tag{3}$$

Here, $f^*(c) := \min\{f(x,c) : x \in \mathcal{X}\}$ is the best possible objective value with respect to scenario $c \in \mathcal{U}$ and is used as a benchmark for any other solution $x \in \mathcal{X}$. Both approaches evaluate the robustness of a solution only based on its (absolute or relative) worst-case performance in the objective.

An *ideal minmax regret solution* $x^I \in \mathcal{X}$ is one, where the objective value

$$\max_{c\in\mathcal{U}} \left(f(x^I, c) - f^*(c)\right) = 0 \tag{4}$$

of *MMR* is equal to 0, which means that some $x^I \in \mathcal{X}$ can be found which is optimal for each scenario $c \in \mathcal{U}$. Although an ideal minmax regret solution is highly desirable, one can, in general, not expect to find such a solution. We, therefore, propose in this paper a modified version, the *ranking robust counterpart* which relaxes the condition of an ideal minmax regret solution to

$$\max_{c\in\mathcal{U}} \left(f(x^{RR}, c) - f(x^K(c), c)\right) = 0 \tag{5}$$

where $x^K(c)$ is a *K* best solution of $P(c)$ in (1).

As a numerical example, consider the following minimization problem with two scenarios c_1 and c_2, and three solutions *A*, *B*, and *C*. The objective values are given in Table 1.

Solution *A* has the best worst-case performance, and is the optimal solution to *MM*. However, it ignores the poor performance of *A* compared to *B* and *C* in scenario c_1. Solution *C* has the smallest maximum regret, and is the optimal solution to *MMR*. Solution *B* is the second-best solution in every scenario, and is thus also interesting as a compromise solution from a practical perspective (while both *A* and *C* can be the worst choices in one of the scenarios, respectively).

Alternatively, we may also consider each scenario as an objective function of a multi-criteria optimization problem. There usually does not exist a single solution that performs best for all objective functions at the same time; instead, one aims at finding Pareto solutions (see Ehrgott, 2006). It can be shown that the set of Pareto solutions also includes optimal solutions to *MM* and *MMR* (Aissi et al., 2009).

Choosing one solution out of the set of Pareto solutions is already a difficult task that is hard to automate, as it depends on the practical insight and priorities of the decision maker (see Miettinen, 2014 for a survey on visualization methods that guide such a selection process). One approach to select such a desired solution from the set of candidates is to roughly classify their performance in each objective, and to choose one that never falls into a bottom-percentile performance class. Such an approach also leads to our concept of ranking robustness.

Our method is related to the robust optimization approach presented in Buhmann, Mihalák, Srámek, and Widmayer (2013). For any $\rho \geq 1$, the authors consider the set of ρ-approximate solutions in each scenario. Their aim is to find ρ large enough, such that the intersection of these sets is non-empty. Furthermore, a value for ρ is to be found which maximizes what they call the unexpected similarity between the solution sets.

In the following, we formalize our approach of ranking robustness. We introduce a general definition for ranking robust optimization problems and discuss general properties in Section 2. We then consider two variants in more detail: solution ranking robustness in Section 3, and objective ranking robustness in Section 4. These approaches are compared in Section 5, and applied to the shortest path problem in Section 6. A computational example applying our approach to the shortest path problem on a real-world street network in the context of evacuation planning is presented in Section 7, before the paper is concluded in Section 8.

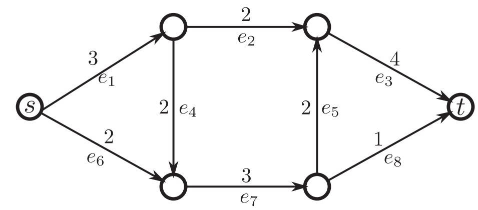

Fig. 1. A shortest path instance.

Table 2
Feasible solutions to the example shortest path instance from Fig. 1.

Path name	Path	Length
P_1	(e_1, e_2, e_3)	9
P_2	$(e_1, e_4, e_7, e_5, e_3)$	14
P_3	(e_1, e_4, e_7, e_8)	9
P_4	(e_6, e_7, e_5, e_3)	11
P_5	(e_6, e_7, e_8)	6

2. Ranking robustness

We consider combinatorial optimization problems

$$(P) \qquad \min\{f(x,c) = c^t x : x \in \mathcal{X}\} \tag{6}$$

over some set $\mathcal{X} \subseteq 2^E$ of feasible solutions, where $E = \{e_1, \ldots, e_m\}$ is a finite ground set (equivalently $\mathcal{X} \subseteq \mathbb{B}^m$ and $x \in \mathcal{X}$ a binary vector). Due to data uncertainty, we assume that the cost coefficients c are not known exactly, but are known to stem from some set of possible outcomes $\mathcal{U}$, also called the uncertainty set. We write $P(c)$, $c \in \mathcal{U}$ to denote that problem *P* is uncertain and depending on *c*.

Inspired by ranking problems (often also referred to as *K* best problems, see, e.g., Hamacher & Queyranne, 1985), we introduce the following notation.

Definition 1. For each $c \in \mathcal{U}$ a *priority list (with respect to c) with length L(c)* is an ordered partition of the set $\mathcal{X}$ of feasible solutions into $L(c)$ subsets, i.e.,

$$\mathcal{S}(c) = \left(S_1(c), S_2(c), \ldots, S_{L(c)}(c)\right) \quad \text{with}$$

$$\bigcup_{i=1}^{L(c)} S_i(c) = \mathcal{X},\; S_i(c) \cap S_j(c) = \emptyset$$

for $i \neq j$, $i, j \in \{1, \ldots, L(c)\}$.

Definition 2. Given $c \in \mathcal{U}$ and a priority list $\mathcal{S}(c)$, $x \in S_i(c)$ is said to be *preferred to* $y \in S_j(c)$ iff $i < j$. For $x \in S_i(c)$ and $y \in S_j(c)$ with $i < j$, we say that *x* is preferred to *y* in scenario $c \in \mathcal{U}$.

Generally speaking, a priority list should encapsule the preferences of a decision maker under each scenario. Hence, there may be different approaches to construct such lists. We illustrate some in the following.

Example 1. We consider the shortest $s - t$ path instance from Fig. 1. Next to each edge, its name and length are shown. Table 2 summarizes all feasible solutions in this setting.

One natural approach to construct a priority list is to group paths according to their objective ranking, i.e., their total length,

Please cite this article as: M. Goerigk et al., Ranking robustness and its application to evacuation planning, European Journal of Operational Research (2016), http://dx.doi.org/10.1016/j.ejor.2016.05.037