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A biologically inspired solution for fuzzy shortest path problems

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ABSTRACT

By considering the uncertainty that exists in the edge weights of the network, fuzzy shortest path problems, as one of the derivative problems of shortest path problems, emerge from various practical applications in different areas. A path finding model, inspired by an amoeboid organism, *Physarum polycephalum*, has been shown as an effective approach for deterministic shortest path problems. In this paper, a biologically inspired algorithm called Fuzzy *Physarum* Algorithm (*FPA*) is proposed for fuzzy shortest path problems. FPA is developed based on the path finding model, while utilizing fuzzy arithmetic and fuzzy distance to deal with fuzzy issues. As a result, *FPA* can represent and handle the fuzzy shortest path problem flexibly and effectively. Distinct from many existing methods, no order relation has been assumed in the proposed *FPA*. Several examples, including a tourist problem, are given to illustrate the effectiveness and flexibility of the proposed method and the results are compared with existing methods.

1. Introduction

The shortest path problem is one of the most fundamental problems in a wide variety of applications, such as road navigation in transportation systems [1,2], traffic routing in communication networks [3,4], and path planning in robotic systems [5,6] and so on [7,8]. The objective of this problem is to find the shortest path in a network from a pre-determined starting node to a pre-determined ending node. Conventionally, the weight of each edge, representing time or cost in traversing the edge, is assumed to be crisp. However, in practical applications, it is difficult for decision makers to determine the weights. For example in transportation, the actual geographical distance of each road may be known deterministically, however, the cost or time in travelling through the road may fluctuate with traffic conditions, weather. A typical way to handle uncertainties in edge weights is to use fuzzy set theory, which has been widely studied and applied in various fields [9–12].

The problem of finding the shortest path in a network with fuzzy edge weights is known as the fuzzy shortest path problem (FSPP), which was first analyzed in 1980 by Dubois and Prade [13] and solved by considering the extensions of the classic Floyd and Ford-Moore-Bellman algorithms. The major drawback of the method is that the path derived by the algorithms may not actually exists in the network. Since then, numerous studies have considered this problem [14–16]. Lin and Chern [17] defined most vital arcs in the network and derived the membership function of the shortest distance by means of a fuzzy linear programming approach. By representing edge weights as interval numbers, Okada and Gen [18] proposed a generalization of Dijkstra's algorithm to find the shortest path. In [19], a hybrid intelligent algorithm based on the genetic algorithm and fuzzy simulations was developed for three fuzzy programming models, which are expected shortest path model, the most shortest path model and α-shortest path model. With studies in order relations between fuzzy numbers developing, many approaches based on order relations were proposed for fuzzy shortest path problems [20–25]. Based on "fuzzy min" order relation, Okada and Soper [20] developed an algorithm to obtain a set of nondominated paths or Pareto optimal paths. Then a new comparison index was defined by considering interactivity among fuzzy numbers [21]. In [22], the Yager ranking indices were used to transform fuzzy numbers into crisp numbers to find the shortest path in network flow problems. Hernandes et al. proposed an iterative algorithm based on the Ford-Moore-Bellman algorithm with a generic ranking index, in which different order relations can be chosen by decision-maker according to concrete problems [23].

Recently, an amoeboid organism, *Physarum polycephalum*, was found to be capable of finding the shortest path [26–30]. By extracting the underlying physiological mechanism of its path finding ability, a path finding mathematical model was constructed [31]. It was shown that the model is effective in finding the shortest path

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and road navigation in complex road networks [32,33]. Furthermore, it was found that the plasmodium has the ability of forming networks with properties comparable to or better than the Tokyo rail network [34].

In this paper, we are motivated to develop a biologically inspired algorithm for fuzzy shortest path problems. Since the original path finding model [31] inspired by *P. polycephalum* is only designed for classical shortest path problems, it is not applicable under fuzzy environments. Therefore, in this paper we modify the algorithm with the help of fuzzy set theory. At first, fuzzy numbers are used to represent the fuzzy shortest path problem. Then, fuzzy arithmetic and fuzzy distance between fuzzy numbers are adopted to modify the path finding algorithm, resulting in the proposed Fuzzy *Physarum* Algorithm (*FPA*). It is found that *FPA* can represent and handle the fuzzy shortest path problem flexibly and effectively. In addition, distinct from many existing methods, one feature of *FPA* is that no order relation between fuzzy numbers is used.

The rest of the paper is organized as follows. In Section 2, some basic concepts and theories are reviewed. Section 3 describes the proposed *FPA* for solving the fuzzy shortest path problem. Some examples, including a tourist problem, are given to demonstrate the performance of our proposed algorithm in Section 4. Finally, concluding remarks are given in Section 5.

2. Definitions and preliminaries

In this section, some basic definitions used throughout the paper are briefly reviewed from [35,9,36,25,37], including fuzzy numbers, fuzzy arithmetic and fuzzy distance.

2.1. Fuzzy numbers

Fuzzy numbers are widely used in statistics, computer programming, engineering, experimental science and so on, in order to represent vagueness and lack of precision of data [38–44]. The theory of fuzzy numbers is based on the theory of fuzzy sets, proposed by Zadeh [9] in 1965. The concept of a fuzzy number was first used by Nahmias in the United States and by Dubois and Prade in France in the late 1970s. The most commonly used shapes of fuzzy numbers are triangular and trapezoidal. Their definitions given by [35,37], along with some basic notions on fuzzy sets, are given as follows.

Definition 1. A fuzzy set $\widetilde{A}$ defined on a universe X may be expressed as:

$$\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x)\rangle | x \in X\} \tag{1}$$

where $\mu_{\widetilde{A}} \to [0, 1]$ is the membership function of $\widetilde{A}$. The membership value $\mu_{\widetilde{A}}(x)$ describes the degree of $x \in X$ in $\widetilde{A}$.

Definition 2. A fuzzy set $\widetilde{A}$ of X is normal iff $\sup_{x\in X}\mu_{\widetilde{A}}(x) = 1$.

Definition 3. A fuzzy set $\widetilde{A}$ of X is convex iff $\mu_{\widetilde{A}}(\lambda x + (1-\lambda)y) \geq (\mu_{\widetilde{A}}(x) \wedge \mu_{\widetilde{A}}(y))$, $\forall x, y \in X, \forall \lambda \in [0, 1]$, where $\wedge$ denotes the minimum operator.

Definition 4. A fuzzy set $\widetilde{A}$ is a fuzzy number iff $\widetilde{A}$ is normal and convex on X.

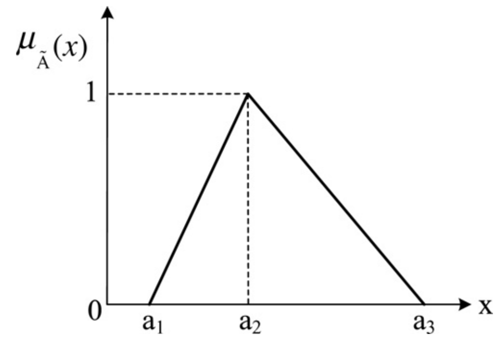

Fig. 1. A triangular fuzzy number $\widetilde{A}$.

Definition 5. A triangular fuzzy number $\widetilde{A}$ is a fuzzy number with a piecewise linear membership function $\mu_{\widetilde{A}}$ defined by:

$$\mu_{\widetilde{A}} = \begin{cases} 0, & x \leq a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & a_3 \leq x \end{cases} \tag{2}$$

which can be denoted as a triplet (a_1, a_2, a_3). A triangular fuzzy number $\widetilde{A}$ in the universe set X that conforms to this definition shown in Fig. 1.

Definition 6. A trapezoidal fuzzy number $\widetilde{A}$ is a fuzzy number with a membership function $\mu_{\widetilde{A}}$ defined by:

$$\mu_{\widetilde{A}} = \begin{cases} 0, & x \leq a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_3-a_2}, & a_4 \leq x \leq a_3 \\ 0, \quad a_4 \leq x \end{cases} \tag{3}$$

which can be denoted as a quartet (a_1, a_2, a_3, a_4). A trapezoidal fuzzy number $\widetilde{A}$ in the universe set X that conforms to this definition is shown in Fig. 2.

2.2. Fuzzy arithmetic

Fuzzy arithmetic on both triangular and trapezoidal fuzzy numbers used in this paper is shown as follows [45,46].

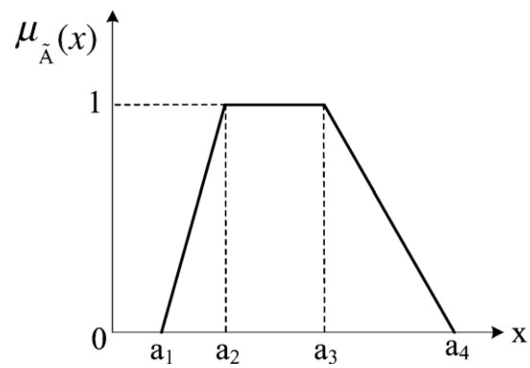

Fig. 2. A trapezoidal fuzzy number $\widetilde{A}$.