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Interfaces with Other Disciplines

Optimal construction and rebalancing of index-tracking portfolios

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ABSTRACT

Index funds aim to track the performance of a financial index, such as, e.g., the Standard & Poor's 500 index. Index funds have become popular because they offer attractive risk-return profiles at low costs. The index-tracking problem considered in this paper consists of rebalancing the composition of the index fund's tracking portfolio in response to new market information and cash deposits and withdrawals from investors such that the index fund's tracking accuracy is maximized. In a frictionless market, maximum tracking accuracy is achieved by investing the index fund's entire capital in a tracking portfolio that has the same normalized value development as the index. In the presence of transaction costs, which reduce the fund's capital, one has to manage the trade-off between transaction costs and similarity in terms of normalized value developments. Existing mathematical programing formulations for the index-tracking problem do not optimize this trade-off explicitly, which may result in substantial transaction costs or tracking portfolios that differ considerably from the index in terms of normalized value development. In this paper, we present a mixed-integer linear programing formulation with a novel optimization criterion that directly considers the trade-off between transaction costs and similarity in terms of normalized value development. In an experiment based on a set of real-world problem instances, the proposed formulation achieves a considerably higher tracking accuracy than state-of-the-art formulations.

1. Introduction

An investment fund is a professionally managed investment vehicle that pools funds from different investors to invest in a portfolio of securities such as stocks or bonds. Over the past decade, a specific type of investment fund, so-called index funds, have become increasingly popular. At the end of 2015, over US-$2 trillion were invested in US-based index funds (cf. Investment Company Institue, 2016). Index funds aim to track the future value development over a given investment horizon of a benchmark portfolio, which represents an investment of the fund's entire capital in a financial index, such as the Standard & Poor's 500 index. Index funds require less investment research and often yield higher net returns compared to the investment funds that aim to outperform the benchmark portfolio (cf., e.g., Busse, Goyal, & Wahal, 2010; Malkiel, 1995; Montfort, Visser, & van Draat, 2008). The investors of an index fund demand a high tracking accuracy, i.e., small deviations between the future value developments of the benchmark portfolio and the index fund. In a frictionless market, the tracking accuracy is maximized when the index fund invests its entire capital in a tracking portfolio that has the same normalized shape as the index. The normalized shape refers to the shape of the value development that is normalized to a value of one today. The most intuitive index-tracking approach is to invest in all index constituents according to the index composition, which is known as full replication. Full replication ensures that the normalized shapes of the tracking portfolio and the index coincide. However, in the presence of market frictions, full replication causes a substantial amount of transaction costs that reduce the fund's capital, especially for indices with many constituents (cf., e.g., Beasley, Meade, & Chang, 2003). This reduction of the fund's capital negatively impacts the tracking accuracy. Often, a higher tracking accuracy can be obtained by balancing the trade-off between the transaction costs and the similarity in terms of normalized shape.

The index-tracking problem considered in this paper consists of revising (rebalancing in the following) the composition of the index fund's tracking portfolio in response to new market information and cash changes that occur due to deposits and withdrawals from investors. Purchasing or selling assets to rebalance the tracking portfolio causes fixed and proportional transaction costs. The objective is to maximize the tracking accuracy subject to various practical portfolio constraints, including minimum transaction values that are imposed by many stock exchanges and short-selling restrictions that prohibit selling securities not currently owned by the fund. The initial construction of the tracking portfolio is a special case of the index-tracking problem in which the existing

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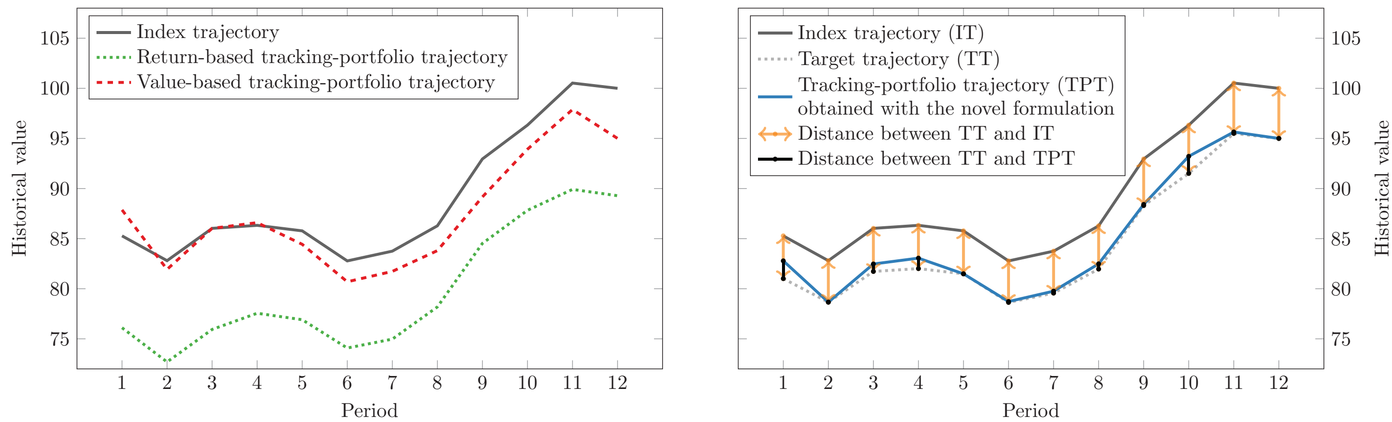

(a) Existing formulations: large distance or different shapes

(b) Novel formulation: small distance and similar shapes

Fig. 1. Drawbacks of existing formulations and advantages of the proposed formulation.

tracking portfolio consists of cash only. In practice, index-fund managers solve the index-tracking problem periodically over the fund's lifetime, which allows to account for changed market environments and cash changes.

Various mathematical programing formulations for the index-tracking problem have been proposed in the literature. These formulations determine a tracking portfolio by minimizing an objective function, referred to as tracking error, that measures the difference between the historical performance of the tracking portfolio and the index. The underlying idea is that, even though a low tracking error in the past does not guarantee a high tracking accuracy in the future, tracking portfolios with a lower tracking error tend to have a higher tracking accuracy in the future. Based on the specific tracking-error function used, existing formulations are divided into the two classes value-based formulations and return-based formulations (cf., Gaivoronski, Krylov, & Van der Wijst, 2005). The value-based formulations measure the tracking error as the distance between the historical trajectories (value developments) of the tracking portfolio and the index (cf., e.g., Guastaroba & Speranza, 2012; Konno & Wijayanayake, 2001). The values of the historical index trajectory correspond to the values of the index scaled by a constant factor such that the value at the end of the last historical period is equal to the capital of the index fund. The end of the last historical period corresponds to the point in time when the portfolio is rebalanced. The return-based formulations measure the tracking error as a function of the differences between the historical returns of the tracking portfolio and the index (cf., e.g., Andriosopoulos & Nomikos, 2014). Intuitively, minimizing this return-based tracking error can be viewed as maximizing the similarity between the normalized shapes of the historical index trajectory and the historical tracking-portfolio trajectory. In the presence of transaction costs, both classes of formulations determine tracking portfolios with an unsatisfactory tracking accuracy. Return-based formulations tend to determine tracking portfolios whose historical normalized shape is similar to the historical normalized shape of the index, but whose historical trajectory lies considerably below the historical index trajectory because the distance between the trajectories is not penalized in the objective function (cf. Fig. 1a). Value-based formulations tend to determine tracking portfolios whose historical trajectories are overall closer to the historical trajectory of the index, but whose normalized shapes differ considerably from the normalized shape of the index (cf. Fig. 1a). By deviating from the normalized shape of the index, the tracking portfolio can reduce the distance between its trajectory and the trajectory of the index that is inevitably caused by transaction costs. However, to achieve a high tracking accuracy, both a small distance between the trajectories and similar normalized shapes are required. Another disadvantage of existing return-based and value-based formulations is that they lead to substantial rebalancing costs when applied periodically over the fund's lifetime. Return-based formulations rebalance heavily because the associated transaction costs are not penalized in the objective function. Value-based formulations lead to substantial rebalancing because the rebalancing costs affect the distance between the trajectories only in the last historical period and can be overcompensated by a reduced distance in the previous periods. The above mentioned drawbacks can be addressed by combining value-based and return-based tracking-error measures in a single optimization criterion and limiting the rebalancing costs. However, since return-based formulations calculate the historical returns of the tracking portfolio either by using a non-convex function (cf., e.g., Beasley et al., 2003) or by assuming constant weights of the components in the tracking portfolio over time (cf., e.g., Canakgoz & Beasley, 2009), the resulting optimization criterion would represent a non-convex function or would rely on simplifying assumptions. Moreover, weighting the two measures and choosing an appropriate limit on the rebalancing costs would require an instance-specific fine tuning by the user.

In this paper, we propose a new value-based mixed-integer linear programing (MILP) formulation. The main feature of this formulation is that it determines a target trajectory internally instead of using the historical index trajectory as done by existing value-based formulations. The target trajectory has the same returns as the index, but its level may vary. As shown in Fig. 1b, the objective function then minimizes both, the distance between the tracking-portfolio trajectory and the target trajectory, and the distance between the target trajectory and the index trajectory. Minimizing the former distance can be viewed as maximizing a similarity measure between the normalized shape of the tracking portfolio and the normalized shape of the index. In contrast to return-based formulations, which use a different similarity measure, the proposed similarity measure can be formulated as a linear function without assuming constant portfolio weights over time. Minimizing the latter distance leads to tracking portfolios with low rebalancing costs. Minimizing both distances simultaneously corresponds to optimizing the trade-off between rebalancing costs and similarity in terms of normalized shape. The proposed formulation represents a generalization of existing value-based formulations because if the distance between the target trajectory and the index trajectory is forced to be zero, e.g., when transaction