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Interfaces with Other Disciplines

Geometric composite indicators with compromise Benefit-of-the-Doubt weights

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ABSTRACT

This paper builds on Van Puyenbroeck and Rogge's (2017) 'indirect' multiplicative Benefit-of-the-Doubt (BoD) index number framework, in which the linear, data-driven BoD-model is used to estimate the importance of various sub-indicators within a geometric composite index (CI). We present an integrated framework that combines optimistic and pessimistic BoD-based weighting that enables to (1) establish the degree of unbalance in countries' policy portfolio mix, (2) identify multiple underlying factors to explain inter-temporal evolution, and (3) explain for differences in country policy performances under the different weighting schemes following a multiplicative Bortkiewicz decomposition. In doing so, we use alternative optimistic and pessimistic BoD-models as existing models suffer from potential drawbacks in the identification of a country's comparative strengths and weaknesses. We illustrate our results with social inclusion data for the EU-countries for the period 2008-2013.

1. Introduction

The flexibility and the optimistic stance in the determination of the weights is often praised as the most important advantage of the Benefit-of-the-Doubt (BoD) method in the construction of composite indicators (CIs).[1] In a setting in which objective knowledge on the true policy weights is usually lacking or incomplete, the BoD-model derives for each country the set of optimal weights from the observed sub-indicator values themselves. More in particular, the BoD-model defines importance weights for each country such that the impact of sub-indicators of relative strength is maximized and the impact of sub-indicators of relative weakness is minimized in the composite value. This quality explains much of the appeal of the BoD-model: in what is usually a sensitive evaluation environment, disappointed countries can no longer blame a low CI-score on damaging or unfair weights. Any other weighting scheme than the one specified by the BoD-model would worsen the CI-score. The increasingly popular BoD-model has by now become an established method to construct CIs in various contexts.[2]

In recent years, several interesting extensions of the BoD-model have been proposed in the literature. One such extension is the so-called "pessimistic" version of the BoD-model that relates to the minimum efficiency concept introduced by Zhu (2004) in the DEA-context. The conceptual starting point of the pessimistic counterpart is opposite to the one of the traditional, 'optimistic' BoD-model (Rogge, 2012; Zhou, Ang, & Poh, 2007). Specifically, the pessimistic BoD-model assesses the policy performance of countries under a 'worst-case' evaluation scenario, in which weights for the sub-indicators are defined such that the CI-value of each country is minimized relative to the other countries. In the definition of the weights this entails assigning relatively high (low) weights to sub-indicators on which the evaluated country performs relatively weakly (strongly) as compared to the other countries in the sample.

Other interesting extensions are the multiplicative versions of the BoD-model which compute CIs by a multiplicative aggregation function. Multiplicative CIs do not imply, contrary to their linear equivalents, full compensability which makes that a poor performance on one sub-indicator cannot be fully compensated by

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[1] The BoD-technique is inspired by (the multiplier formulation of) Data Envelopment Analysis (DEA) (Charnes, Cooper & Rhodes, 1978), an efficiency measurement technique popular in the Management Science and Operations Research literature.

[2] Examples include human development (Blancard & Hoarau, 2013), environmental performance (Rogge, 2012), measuring active ageing (Amado, São José & Santos, 2016), measuring local police effectiveness (Verschelde & Rogge, 2012), measuring quality in health care (Shwartz, Burgess & Zhu, 2016).

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sufficiently high values on other sub-indicators (see, e.g., Ebert & Welsch, 2004). In addition, multiplicative CIs penalize inequality among sub-indicators (Nardo, Saisana, Saltelli, & Tarantola, 2008, p. 33). In the BoD-literature, there are, broadly speaking, two streams of multiplicative BoD-models.[3] A first stream of 'direct' multiplicative BoD-models combine both multiplicative aggregation and BoD-weighting in one computation step (Blancas, Contreras, & Ramírez-Hurtado, 2012; Tofallis, 2014; Zhou, Ang, & Zhou, 2010). More recently, Van Puyenbroeck and Rogge (2017) proposed an alternative, 'indirect' approach to construct multiplicative BoD-based CIs. It concerns a two-step procedure in which, in a first step, importance weights of the different sub-indicators are estimated using the original BoD-model and, in a second step, BoD-derived importance weights are used in the construction of the CIs as geometric mean quantity indices. It is argued by Van Puyenbroeck and Rogge (2017) that this 'indirect' multiplicative BoD-model avoids some of the disadvantages in the 'direct' multiplicative BoD-models.[4]

This paper contributes to the literature in three ways. Firstly, the paper extends the indirect multiplicative BoD-model of Van Puyenbroeck and Rogge (2017) by combining optimistic and pessimistic BoD-weighting so as to obtain a comprehensive view on countries' policy performances.[5] A second contribution of the paper is more innovative and relates to the 'optimistic' and 'pessimistic' BoD-models that are used in the extension of the framework of Van Puyenbroeck and Rogge (2017). In particular, as existing versions of the optimistic and pessimistic BoD-models may cause implausible results in the identification of a country's sub-indicators of comparative strength and weakness, we advocate new versions of optimistic and pessimistic BoD-weighting. These alternative BoD-weighting models are different in the sense that the benchmark is a fixed, hypothetical country. It is shown that by evaluating each country in the sample set relative to a fixed benchmark, sub-indicators can no longer be simultaneously identified as a comparative strength and weakness. A third original contribution is the development of a measure for the degree of unbalance in a country's policy portfolio mix. It concerns a ratio of geometric CI under optimistic weighting and the geometric CI under pessimistic weighting that can be further decomposed using a multiplicative Bortkiewicz (Bortkiewicz, 1924) decomposition to explain for (changes in) the degree of unbalance in a country's policy portfolio mix.

The paper is structured as follows. Section 2 briefly describes Van Puyenbroeck and Rogge's (2017) 'indirect' framework for multiplicative BoD-based CI-construction. This section also illustrates some important issues with the existing optimistic and pessimistic BoD-models in the identification of countries' comparative strengths and weaknesses. To resolve for these issues, we present alternative versions of the optimistic and pessimistic BoD-weighting models and implement them into the 'indirect' framework to derive a compromise geometric CI. In Section 3, we adjust Van Puyenbroeck and Rogge's (2017) inter-temporal geometric CI-framework, so as to incorporate both optimistic and pessimistic BoD-based weights. Section 4 develops a measure for the degree of unbalance in a country's policy portfolio mix. This section also shows how this measure can be decomposed using a multiplicative Bortkiewicz decomposition to explain for (changes in) the degree of unbalance in a country's policy portfolio mix. Section 5 concludes.

Throughout, we illustrate our findings with the commonly agreed EU indicators (period 2008–2013) from the overarching portfolio of social protection and social inclusion objectives as endorsed by the Heads of State and Government in the Europe 2020-strategy and employed by Social OMC (Social Protection Committee, 2015). Specifically, the nine overarching commonly agreed EU social inclusion indicators are: (i) *at risk of poverty or social exclusion rate*, (ii) *relative median poverty risk gap*, (iii) *income quintile ratio (S80/S20)*, (iv) *early school leavers*, (v) *aggregate replacement ratio*, (vi) *at-risk-of-poverty rate anchored at a fixed moment in time (2008)*, (vii) *employment rate of older workers*, (viii) *in work at-risk-of poverty rate*, and (ix) *activity rate.* For all except three indicators (i.e., the aggregate replacement ratio, the employment rate of older workers and the activity rate), higher values represent worse social inclusion performances. To put all indicators on a common basis so that all measure social inclusion, the other six indicators are transformed by taking the inverse of the regular indicator.

2. Compromise geometric mean composite indicators

The indirect CI-framework of Van Puyenbroeck and Rogge (2017) computes multiplicative CIs as geometric mean quantity index numbers using BoD-derived sub-indicator importance weights by a two-step procedure. In a first step, importance weights for the different sub-indicators are estimated using the original (linear) BoD-model. In a second step, (normalized) country sub-indicator values are weighted and geometrically aggregated using the BoD-based importance weights as obtained in the first step. Formally,

$$CI_i = \prod_{r=1}^{s} \left(\frac{y_{r,i}}{y_{r,B}} \right)^{\omega_{r,i}} \tag{1}$$

where $y_{r,i}$ in the numerator is the performance of the ith country ($i = 1,\ldots,n$) on the rth social inclusion sub-indicator ($r = 1,\ldots,s$). In the denominator, there are the baseline sub-indicator values, $y_{r,B,}$ relative to which the performances of country i are compared (in our application below, the baseline performance values are equal to the (population-weighted aggregate) sub-indicator values for the EU-27 countries, i.e., $y_{r,B} = y_{r,EU27}$).[6] The sub-indicator exponents $\omega_{r,i}$ defines how much the rth sub-indicator contributes to the aggregate CI, with $\sum_{r=1}^{s} \omega_{r,i} = 1$. The sub-indicator exponent values $\omega_{r,i}$ indicate the percentage change in the CI_i-value as result of a 1% increase in $\frac{y_{r,i}}{y_{r,B}}$. Note that the multiplicative CIs as in (1) are tailor-made per country to compare the evaluated country itself with some base performance observation. As such, they are bilateral in nature and, hence, should be interpreted in relative terms. For example, a CI_i -value of 1.1 indicates that the policy performance of the country i on the whole outperforms the baseline policy performance by 10%. A CI_i -value of 0.8, on the other hand, indicates that the policy performance of the country i on the whole underperforms the baseline policy performance by 20%.

In their original work, Van Puyenbroeck and Rogge (2017) use the traditional optimistic (linear) BoD-model (see Model A1 in Appendix A) to estimate the exponents $\omega_{r,i}$ of the geometric

[3] Both 'direct' (Giambona & Vassallo, 2014) and 'indirect' (Van Puyenbroeck & Rogge, 2017) versions of the multiplicative BoD-model have been applied on European social inclusion data in recent studies.

[4] The 'disadvantages' of the direct multiplicative BoD-models include the lack of commensurability (as in the model of Zhou et al., 2010) and the presence of a scaling factor as in the model of Tofallis (2014) (for a more elaborate discussion of the disadvantages of the direct multiplicative BoD-model, see Van Puyenbroeck & Rogge, 2017).

[5] Note that the idea of combining 'optimistic' and 'pessimistic' weighting scenarios in the indirect BoD-CI framework so as to obtain a comprehensive view on countries' policy performances was suggested by Van Puyenbroeck and Rogge (2017) in the concluding section of their paper.

[6] As noted by Van Puyenbroeck and Rogge (2017), the choice of a specific set of base performance indicators $y_{r,B}$ is largely arbitrary. Depending on the evaluation context, base performance values other than the sample average of each sub-indicator can be specified (e.g., median, maximum, etc.). Within the EU social policy setting, benchmarking performances to the EU27-average was endorsed by the European Commission in its yearly Joint Employment Report.