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Discrete Optimization

A hybrid breakout local search and reinforcement learning approach to the vertex separator problem

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ABSTRACT

The Vertex Separator Problem (VSP) is an NP-hard problem which emerges from a variety of important domains and applications. In this paper, we present an improved Breakout Local Search for VSP (named BLS-RLE). The distinguishing feature of this approach is a new parameter control mechanism that draws upon ideas from reinforcement learning theory to reach an interdependent decision on the number and on the type of perturbation moves. The mechanism complies with the principle of first carrying out intensification and then employing minimal diversification only if needed, it uses a dedicated sampling strategy for a rapid convergence towards a limited set of parameter values that appear to be the most convenient for the given state of search. Extensive experimental evaluations and statistical comparisons on a wide range of benchmark instances show significant improvement in performance of the proposed algorithm over the existing BLS algorithm for VSP. Indeed, out of the 422 tested instances, BLS-RLE was able to attain the best-known solution in 93.8% of the cases, which is around 20% higher compared to the existing BLS. In addition, we provide detailed analyses to evaluate the importance of the key elements of the proposed method and to justify the degree of diversification introduced during perturbation.

1. Introduction

Let $G = (\mathcal{V}, \mathcal{E})$ be an undirected graph where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ is the set of vertices with nonnegative weights $c_1, c_2, \ldots, c_n$ and let $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ be a set of unweighted edges. A vertex separator in a graph is a set of vertices whose removal breaks the graph into two non-empty disconnected components. More formally, the Vertex Separator Problem (VSP) aims to find a partition of $\mathcal{V}$ into disjoint vertex subsets A, B, C with A and B non-empty, such that (i) there is no edge $(i, j) \in \mathcal{E}$, $i \in A$, $j \in B$; (ii) $\max\{|A|, |B|\} \leq b$, $1 \leq b \leq |\mathcal{V}|$ (b is the problem input) and; (iii) $\Sigma\{c_j\colon j \in C\}$ is minimized. A and B are called the shores of the separator C. A separator C is a legal (feasible) solution if it satisfies the problem constraints (i) and (ii), and is termed optimal if (i), (ii) and (iii) are met. A separator C is balanced if $\max\{|A|, |B|\} \leq 2|\mathcal{V}|/3$.

The problem of finding minimal balanced separators is NP-hard (Bui & Jones, 1992). It first arose in the context of Very Large Scale Integration (VLSI) design (Leiserson, 1980; Ullman, 1984), and has become popular in several other applications. For instance, in telecommunication networks, a separator determines the capacity and the brittleness of the network (Leiserson, 1980). In bioinformatics and computational biology, separators in grid graphs provide a simplified representation of proteins (Fu & Chen, 2006). The problem also finds an application in cyber security where it can be used to disconnect a largest connected component in a network to prevent a possible spread of an attack. A recent interesting application of VSP is in decoy routing. The aim of decoy routing is to prevent nation-state level Internet censorship by having routers transfer traffic to blocked destinations. The goal is to cover all paths to a set of destinations which is sufficiently large so that it is not economically viable for a warden to block these destinations. The problem of deploying a minimum number of decoy routers could be related to VSP (Donghyun, Frye, Sung-Sik, Hyung, & Tokuta, 2013; Schuchard, Geddes, Thompson, & Hopper, 2012). Generalization of the vertex separator problem also involves applications in distributed routing protocols (Caria, Jukan, & Hoffmann, 2016). However, perhaps more importantly, finding small balanced separators is a major primitive for many graph algorithms, especially for those that are based on the principle of divide-and-conquer (Bhatt & Leighton, 1984; Evrendilek, 2008; Kayaaslan, Pinar, Çatalyürek, & Aykanat, 2012; Lipton & Tarjan, 1979; Liu, 1989a). For instance, Evrendilek (2008) proposed a novel heuristic to partition a graph by vertex separators in order to

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provide a balanced process of workload and a minimised communication overhead. Vertex separators have also been used for hypergraph partitioning, which has an impact on a wide variety of parallel and distributed computing applications (Kayaaslan et al., 2012).

Several exact algorithms have been proposed for solving VSP (Balas & de Souza, 2005a; 2005b; Biha & Meurs, 2011; Cavalcante & de Souza, 2011). These algorithms are able to find optimal results in a reasonable computing time (within 3 hours) for instances with up to 125 vertices, but fail to solve larger instances. Given the inherent computational complexity of the problem, approximation algorithms (Feige, Hajiaghayi, & Lee, 2005; Feige & Mahdian, 2006; Hager & Hungerford, 2015), heuristic (George & Liu, 1978; Liu, 1989b) and meta-heuristic methods have also been considered. The Breakout Local Search (BLS) algorithm (Benlic & Hao, 2013c), along with a Variable Neighborhood Search (VNS) (Sánchez-Oro, Mladenović, & Duarte, 2014), is among the first metaheuristic approaches applied to the above defined VSP formulation. BLS is a recent variation of the popular Iterated Local Search (ILS) metaheuristic (Lourenco, Martin, & Stützle, 2003) with a particular emphasis on the importance of the perturbation phase. In addition to VSP, BLS has been shown to provide competitive performance for several well-studied combinatorial problems including the quadratic assignment (Benlic & Hao, 2013b) and the maximum clique (Benlic & Hao, 2013a) problems.

Relying on the information related to the search state and progress, the basic idea of BLS is to use a perturbation mechanism to adaptively determine a suitable number and type of perturbation moves for the next round of the perturbation phase. Indeed, the existing BLS algorithm for VSP (Benlic & Hao, 2013c) determines both the number and the type of perturbation moves (random or directed moves) by exploiting information on a number of recently visited local optima stored in a hash table memory structure. Despite its success on several challenging combinatorial problems, this poses a limitation on its search dynamics since a more appropriate degree of diversification could be introduced if these two decisions are reactively determined in an interdependent manner.

The aim of this paper is twofold. Given the importance of VSP, our first objective is to enrich the VSP literature with a new self-adaptive approach, capable of finding high quality solutions with reasonable computing efforts for different types of problem landscapes. The other objective is to investigate how reinforcement learning techniques (Sutton & Barto, 1998) can improve the reactive characteristic of a local search heuristic, i.e., a BLS algorithm. For these purposes, we introduce a hybrid between a Breakout Local Search and a Reinforcement Learning technique (denoted as BLS-RLE), which is an extension of the current BLS for VSP with a new parameter control mechanism based on reinforcement learning. The main novelty of the proposed algorithm is that it adaptively and interdependently determines the values for two important parameters: the number of perturbation moves l and the probability e of using a directed over a random perturbation type. Reinforcement learning techniques for parameter control have recently become a feature of the research agenda of the Evolutionary Computing community (Eiben, Horvath, Kowalczyk, & Schut, 2006; Karafotias, Hoogendoorn, & Eiben, 2015; Müller, Schraudolph, & Koumoutsakos, 2002), and the Local Search community (Battiti, Brunato, & Campigotto, 2008a; Prestwich, 2008). The proposed parameter control mechanism is modeled as the multi-armed bandit problem (Auer, Cesa-Bianchi, & Fischer, 2002; Robbins, 1952), a reduced version of the reinforcement learning problem where learning is performed with respect to a single state. In a multi-armed bandit problem, the agent is repeatedly faced with a choice among different options, or actions. In our case, an action corresponds to the application of a parameter pair configuration (l, e) for the perturbation phase. After each action, the agent receives a numerical *reward* that captures the immediate impact on the search process. While the *reward* points out whether an action is good in an immediate sense, an *action value* is the total amount of reward an agent can expect to accumulate over the future. The objective of the multi-armed bandit problem is to maximize the expected action value over some time period. The originality and success of the proposed parameter controller lie in the two following key elements: (i) a dedicated strategy for sampling of configurations (l, e), which enables rapid convergence towards a limited set of actions (parameter pair settings) that appear to be the most convenient for the given state of search, and (ii) a reward function which draws upon the following idea: "intensification first, minimal diversification only if needed" (Battiti, Brunato, & Mascia, 2008b). Furthermore, the proposed mechanism does not introduce a significant computational overhead compared to the existing BLS for VSP, while substantially improving the performance of BLS. Even though the objective of this work is to provide an effective approach to VSP, the proposed RL controller for BLS is general and can be considered for solving other important combinatorial problem. The corresponding code is available online for future use.

The existing BLS, as well as BLS-RLE, are able to solve to optimality all the instances from the current VSP benchmark (Balas & de Souza, 2005a) within less than a second (Benlic & Hao, 2013c). To evaluate the performance of BLS-RLE with respect to BLS and several other algorithms from the ILS family, we thus use a new set of 426 challenging instances with different sizes and structures. A large number of these instances are motivated by several VSP applications including VLSI design, cyber security, and decoy routing. Extensive computational comparisons highlight the benefit of the proposed parameter control mechanism. Indeed, BLS-RLE significantly improves the performance of BLS and competes very favorably with several other variants of ILS. Briefly, out of the 422 considered instances, BLS-RLE was able to attain the best-known solution in 93.8% of the cases, which is around 20% higher compared to the BLS algorithm.

The rest of the paper is organized as follows. In Section 2, we provide a motivation and the general framework of the existing BLS method for combinatorial problems. Section 3 presents the improved BLS framework and applies it to the vertex separator problem. Experimental results and extensive statistical comparisons are given in Section 4. Section 5 provides analyses of the problem landscape to evaluate the hardness of the used benchmark instances, and to justify the behavior of the proposed approach. This section further analyzes the importance of the key algorithmic components of BLS-RLE. Finally, conclusions are presented and discussed in Section 6.

2. Breakout local search

Iterated Local Search (ILS) (Lourenco et al., 2003) is a popular metaheuristic which has been used for tackling a large variety of hard combinatorial problems. Its basic idea is to iterate between a local search phase to intensify the search, and a perturbation phase to diversify the search. The degree of diversification introduced by an ILS method depends both on the number and on the type of moves applied for perturbation. For most ILS algorithms, these values are fixed throughout the search thus introducing a constant degree of diversification regardless of the search state. To cope with this limitation, a variation of ILS, named Breakout Local Search (BLS) (Benlic & Hao, 2013a; 2013b; Ghandi & Masehian, 2015), was recently proposed in the literature. BLS puts particular emphasis on the importance of the diversification phase. Based on relevant information on the search state, it adaptively determines an appropriate number of perturbation moves and selects between