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Dynamic theory of losses in wars and conflicts

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ABSTRACT

We present a new theory for the dynamic evolution of losses incurred in combat, which is verified using available published data from WW1, WW2 and later conflicts.

This new dynamic theory updates and revises the original Lanchester-type proportionality assumptions for exchange rates, and unifies military operational and strategic thinking in warfare with the evolution of human learning as observed in all modern technological systems. The theory is tested using dynamic data from the vast battles of Kursk, Ardennes and the North Atlantic. The loss and exchange rates closely follow the trends for event rates given by the previously established Universal Learning Curve, while indicating differences between the initial attacker and defender. For conflicts from 1865 to 1991, a new correlation has emerged for overall losses as a function of size of force deployed. Lessons learned after WW2 of using the strategy of standoff engagement dramatically reduced losses.

1. Introduction

We have previously demonstrated that the concept of human learning applies to everyday and rare accident and event data, and have extended these ideas to human, software and hardware reliability (Duffey & Ha, 2010; Fiondella & Duffey, 2015). Most recently, for World War 2 (WW2) the same concepts have been successfully applied to U-boat losses and countermeasure effectiveness in the North Atlantic (Duffey & Gallehawk, 2016). This work lead to the present re-examination of the general basis behind previous models developed in the field of operational research for the prediction and reduction of losses in warfare (Blackett, 1943; Morse & Kimball, 1958; Johnson, 1990; Beer, 1994; Hausken & Moxnes, 2005; Lucas and Dinges, 2004). Specifically, we include the effects of varying tactics, strategy and countermeasures during conflicts and battles by both sides.

We present a new theory for the dynamic evolution of losses incurred in combat, which is verified using available published data from World Wars 1 and 2 (WW1, WW2) and later conflicts. The new theory generalizes conflict or combat theory by being based on the concept that the loss rates vary systematically with increasing experience and/or risk exposure during battle itself. The randomness of individual and collective combats on the battlefield emerges as systematic patterns as to which side is winning or losing. The trends and fluctuations depend on the learning and responses on both sides, which are a direct result of both command decision-making and individual human behavior where the observed outcomes are the most likely result of the innumerable battlefield interactions (troops vs. troops, tank vs. tank, tank vs. troops, individual skirmishes, massed assaults etc.). The result is statistically consistent with being the most likely outcome distribution (Duffey & Saull, 2008), and provides a new exponential form for the dynamic loss rate. This new dynamic theory updates and revises the original Lanchester-type proportionality assumptions for exchange rates, and unifies military operational and strategic thinking in warfare with the evolution of human learning as observed in all modern technological systems (Duffey & Saull, 2008).

The analysis of how to achieve superiority in warfare and combat is a massive field of study all to itself, and cannot be repeated here. Deaths and casualties in conflicts and wars are the subject of extensive study, especially to quantify the numerical advantage over the enemy as each seeks to minimize their own relative losses and counter the opponent. After all, if the losses are equal on both sides, we have mutually assured destruction; and if not equal then one side eventually wins. The useful conventional measure of advantage is the average exchange or attrition rate over the conflict, being the ratio of the loss rate incurred by one opponent force (conventionally called “Blue”) to the loss rate inflicted on the other force (conventionally called “Red”).

These exchange or “attrition” ratios, E, have been embodied in the well-known Lanchester equations developed after WW1 and used in WW2, to describe overall losses for air, land and sea battles (Morse & Kimball, 1958; Johnson, 1990; Fricker, 1998; Hausken & Moxnes, 2002; Lucas & Dinges, 2004; Lucas & Turkes, 2004; Tang, 2006), and even applied to studies of insect colonies and regime change. The loss rates on each side are assumed to be

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proportional to the forces and firepower involved, and in the more general correlations there are some five empirical "constants" for any individual battle (Fricker, 1998). Previous results of using such fits to overall and daily battle data have been well summarized elsewhere (Dinges, 2001; Lucas & Dinges, 2004), who found them to be contradictory and anyway not characterized by constant correlation coefficients but are battle or conflict specific. Lucas and Dinges (2004, p 28) state that "*Consequently, despite many efforts, there have been few clear results regarding the validity of Lanchester's equations as a model of aggregate attrition.*"

This extension to a developing or dynamic conflict situation has been examined using the detailed deployment and casualty data available for the Kursk and Ardennes battles during WW2. For the Ardennes battle it has been stated "*Lanchester's basic models do not hold when fit to data from an actual battle*" (Fricker, 1998, p. 19). Specifically, it was shown for the Kursk battle that the dynamic variation during the battle was important (Lucas & Dinges, 2004, p. 17) as follows: "*Much more of the variation in casualties during the Battle of Kursk is explained by the status of the forces considered and the phases of the battle than by the Lanchester variant used. Specifically, we obtain substantially better fits when we use only the forces that are actively fighting. An additional improvement in fit is gained by breaking the battle into its natural phases. Finally, when comparing fits among the basic laws, we observe that Lanchester's linear law fits these aggregate data better than the logarithmic law does and much better than the square law does.*"

In addition, Lucas and Turkes (2004, p. 116) suggest: "*The failure to find any good-fitting Lanchester model suggests that it may be beneficial to look for new approaches to model highly aggregated attrition.*" This statement alone is sufficient motivation and justification for the present study.

Other efforts to generalize the Lanchester relations include time-dependent reinforcement, withdrawal and combat "effectiveness coefficients'" (Protopopescu, Santoro, Dockery, Cox, & Barnes, 1987), and including stochastic exchange or firepower fluctuations via assumed normal, binomial and Poisson distributions (Hausken & Moxnes, 2000; Hausken & Moxnes, 2002). The method importantly allows for time-dependent feedback between the two sides, as one adapts and responds to the other. For the Ardennes battle, Hausken and Moxnes (2002) empirically varied the coefficients for each assumed "phase" (days) of the battle and encompassed some 38 to 68% of the data fluctuations when adopting firepower weighted combat effectiveness multipliers for tanks, armored personnel carriers (APCs), artillery, troops etc. This empirical statistical fitting appears to be a trivial result in that data variations can be covered by uncertainty and probability distributions; and without general guidance on what coefficient variations to use for different situations, Hausken and Moxnes (2002) suggest compiling data for differing war situations and types. This is what is also undertaken in the present work, and we also show that firepower weighting is possibly an unnecessary complication.

Which exchange or attrition relationship or "law" is right or correct remains an important question, and whether and how the exchange rate varies during combat. We develop this key idea of Lucas and Dinges (2004) that the attrition or exchange rate varies during the combat development. Warfare and combat is intrinsically dynamic, changing tactics and deployment as one side strives to take advantage or attack, and the other responds or defends, and vice versa. Battles and conflicts involve *humans*, who manage and control both their weapons and their own actions: an army or force is simply a collection of humans, responding and adapting to the developing situation.

We adopt an entirely new approach to warfare based on the Learning Hypothesis (Duffey & Saull, 2008) as to how humans behave, correct mistakes and make decisions, which is in accord with masses of data for human learning (Anderson, 1990; Fiondella & Duffey, 2015) and with theories in cognitive psychology, like Ohlsson's Theory of Error Correction (Ohlsson, 1996). The commonly used Lanchester equations are then shown to be special limiting cases of this more general result. To prove generality, the theory is validated against data for the battles of Kursk, Ardennes and the North Atlantic in WW2, and important measures are derived for the risk exposure, learning opportunity and strategic countermeasure response. In addition to establishing new general loss rate equations, we also provide new correlations for the casualty data from the US Civil War, WW1, WW2, and for US forces for the Korean, Vietnam and Gulf Wars.

To our knowledge this is the first time that the separate fields of cognitive psychology, physics, engineering reliability, human performance, safety analysis, risk management, and military strategy have been *quantitatively* combined. It underlines the importance of adaptation and flexibility in control and command during combat.

2. The dynamic theory of combat: revising the Lanchester Equations

We examine what learning models would suggest when combatants or opponents *dynamically* change their tactics and strategy as the battle or war proceeds. Warfare is then a dynamic learning situation, as occurs in real combat. Resulting in second order differential relationships, the Lanchester first order equations are then shown to be special cases of the more general result. We can then easily show what the average results will be in terms of comparative loss totals and rates. Some additional mathematical details are given in the Appendix.

In combat or conflicts the idea is to inflict more casualties on the enemy or opposing force. The obvious thought is that the relative losses must be related to the number of casualties and the type of combat or firepower. This idea was captured by the Lanchester equations, where an "exchange rate", E, is defined as the ratio of the rate of losses on the Blue and Red sides for a given battle or risk exposure measure, T. The Blue force strength and losses are N and n, respectively, and M and m for the Red side. The resulting first order differential equation is

$$E = I_n/I_m = \frac{dn}{dT} \Big/ \frac{dm}{dT} = \frac{dn}{dm} \tag{1}$$

Here the symbol T is not necessarily elapsed time, or even conventionally the number of elapsed battle days, but some relevant conflict, experience or risk exposure measure that we define later in Section 4. The definition of Blue "winning" or "conflict advantage" is when E<1 from counting casualties or destroyed combat systems. The loss rates for Blue and Red are strictly called the intensity, I_n or I_m, (Fiondella & Duffey, 2015), which are assumed to be proportional in some way to the force numbers. More general correlations then have forms like (see e.g. Morse & Kimball,1958; Fricker, 1998;Plowes & Adams, 2005;Hausken & Moxnes, 2002),

$$I_n = \frac{dn}{dT} = a^{\gamma}(T)\left[N(T)^p M(T)^q\right] \text{ and}$$
$$I_m = \frac{dm}{dT} = b^{1-\gamma}(T)\left[M(T)^p N(T)^q\right] \tag{2}$$

where the fitting "constants" like γ, a, b, p and q, are adjusted to data for each overall conflict or conventionally for each successive battle day or measure, T. Sometimes sophisticated fitting routines and sampling techniques are used, and assumptions made that the constants vary because of firepower, force concentration and strategy differences (e.g. Hausken & Moxnes, 2002). The expressions in Eq. (2) are *first order* differential equations in n and m, and the *choice* of p and q being 0 or 1 determines if the loss relation is linear, square or logarithmic. Hence, the original Lanchester exchange