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Invited Review

Mathematical optimization approaches for facility layout problems: The state-of-the-art and future research directions

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ABSTRACT

Facility layout problems are an important class of operations research problems that has been studied for several decades. Most variants of facility layout are NP-hard, therefore global optimal solutions are difficult or impossible to compute in reasonable time. Mathematical optimization approaches that guarantee global optimality of solutions or tight bounds on the global optimal value have nevertheless been successfully applied to several variants of facility layout. This review covers three classes of layout problems, namely row layout, unequal-areas layout, and multifloor layout. We summarize the main contributions to the area made using mathematical optimization, mostly mixed integer linear optimization and conic optimization. For each class of problems, we also briefly discuss directions that remain open for future research.

1. Introduction

Facility layout problems (FLPs) are a general class of operations research problems concerned with finding the optimal arrangement of a given number of nonoverlapping indivisible departments within a given facility. The objective is to minimize the total expected cost of inter-departmental flows inside the facility, where the cost incurred for each pair of departments is equal to the rectilinear distance between the centroids of the departments multiplied by their pairwise cost. This cost, generally non-negative, accounts in the aggregate for adjacency preferences as well as costs that may arise from transportation, the construction of a material-handling system, or connection wiring. The facility and the departments are rectangular, and the area of each department is specified, but if the department's dimensions can vary, then determining them is also part of the FLP.

FLPs have a variety of applications. Much of the work was motivated by the physical organization of manufacturing systems, see e.g. Meller and Gau (1996). The FLP is particularly relevant in flexible manufacturing systems that produce an array of different parts. The layout of the production components has a significant impact on the costs and the productivity of these systems, see e.g. Hassan (1994). Other applications of FLPs include balancing hydraulic turbine runners (Laporte & Mercure, 1988), algorithm initialization in numerical analysis (Brusco & Stahl, 2000), VLSI fixed-outline floorplanning (Luo, Anjos, & Vannelli, 2008), and optimal data memory layout generation for digital signal processors (Wess & Zeitlhofer, 2004).

FLPs have been extensively studied in the literature since the 1960s. Numerous variations on the basic problem described above have been considered, and different models have been proposed for each variation. Examples of such variations are: specially structured instances of the problem (e.g. layouts on rows or on loops); dynamic FLPs with time-dependencies; FLPs under uncertainty in the data; and multi-objective FLPs. We refer the reader to the books (Heragu, 2008; Kusiak, 1990) and survey papers (Meller & Gau, 1996; Singh & Sharma, 2006) for more information about the FLP and its variations. A growing collection of FLP benchmark instances is available online (Anjos, 2015).

The FLP is NP-hard in general, so solving it to global optimality in reasonable time is generally difficult. Indeed the restricted version where the dimensions of the departments are all equal and fixed, and the optimization is taken over a fixed set of possible locations for the departments, is known as the quadratic assignment problem, a combinatorial optimization problem well known for its computational difficulty, see e.g. Loiola, de Abreu, Boaventura-Netto, Hahn, and Querido (2007).

The constraints of the basic FLP can be grouped into two sets:

- *Department shape requirements* include the required area, and restrictions on the dimensions (height and width) such as

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bounds on the ratios height/width and width/height, called aspect ratios. These requirements generally lead to convex constraints but still pose some challenges. In particular, requiring small aspect ratios, while desirable in real-world applications, generally makes the problem harder. On the other hand, while the area constraint traditionally required a careful linearization approach, it can be modeled exactly using conic optimization, see e.g. Anjos and Liers (2012).
- *Department location requirements* include the nonoverlap of departments, fitting every department within the facility, assigning certain departments to, or forbidding them from, particular locations within the facility. The main challenge here are the nonoverlap constraints that are inherently nonconvex and combinatorial.

This review is focused on FLPs with the following properties:

1. the departments have different areas
2. the facility can be one-, two-, or three-dimensional.

The different dimensions lead to the three broad classes of FLPs covered in this review, namely row FLPs (Section 2), unequal-areas FLPs (Section 3), and multifloor FLPs (Section 4).

One-dimensional facilities lead to row FLPs, and we categorize them in terms of the number of rows: single-row, double-row, or multi-row. Single-row and double-row problems commonly occur in practical applications, as we discuss in Sections 2.1 and 2.2 respectively. Multi-row problems are a natural extension of the problem to three or more rows, and are considered in Section 2.3.

Unequal-areas FLPs have two-dimensional facilities with a single floor, and we assume that the facility is rectangular and that all the departments fit inside the facility. Unlike in the case of row layouts, not only the position but also the dimensions of each department are optimized. After discussing models and approaches for the basic two-dimensional problem in Sections 3.1–3.4, we consider in Section 3.5 the special case of flexible bay layouts, a type of layout that resembles row FLPs but with the fundamental difference that the width of the bays can vary, depending on the total area of the departments in each bay.

Three-dimensional facilities give rise to multifloor FLPs in which departments are to be placed over two or more floors. This is the focus of Section 4. The survey in Section 4.1 shows that most of the literature proposes models for specific applications rather than for the general problem. For this reason we propose in Section 4.2 a formulation for a generic form of the problem that we hope will motivate further research into multifloor FLP.

Regarding the choice of methodologies, we limit the scope of this review to mathematical optimization-based approaches. These include exact methods, but as the problems increase in difficulty very rapidly, we also include heuristic methods that use mathematical optimization approximations and/or relaxations. While there is a rich literature on heuristic algorithms for FLPs (see e.g. Kothari & Ghosh, 2012; Meller & Gau, 1996; Singh & Sharma, 2006), our focus here is on mathematical optimization approaches, primarily mixed integer linear optimization (MILO), often referred to as mixed integer programing or MIP, semidefinite optimization (SDO), also called semidefinite programing or SDP, and nonlinear optimization. Because of their importance to the success of these approaches, we also include brief discussions of symmetry breaking (Section 5) and valid inequalities (Section 6) as these are essential ingredients for solving the resulting relaxations efficiently.

We conclude with a summary of directions for future research in Section 7.

2. Row FLPs

Row FLPs share the following common problem statement: given a set of rectangular departments each of a given length, a number of rows, and a pairwise non-negative weight for each pair of departments, determine (i) an assignment of departments to rows, and (ii) the positions of the departments in each row, so that the total of the weighted center-to-center distances is minimized. Row FLPs arise in practical contexts where the departments are to be placed in rows with a predetermined separation between the rows due to factors such as the material-handling system or the flows of people. Moreover, within each row, a minimum clearance between departments is needed to satisfy safety and operational requirements. We assume that this clearance is included in the lengths of the departments. We also assume that the rows and the departments all have the same height, that any department can be assigned to any row, and that the distances between adjacent rows are equal. Under these assumptions, solving an instance of the row FLP means resolving three questions:

1. Assign each department to exactly one row;
2. Express mathematically the center-to-center distance between departments (that may or may not be in the same row);
3. Handle possible empty space between departments.

Section 2.1 is concerned with the simplest version of row FLP, namely the single-row FLP. Section 2.2 covers the double-row FLP, and Section 2.3 extends the coverage to the general multirow FLP.

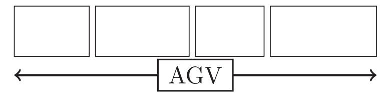

Fig. 1. SRFLP along the path of an AGV.

2.1. The single-row FLP

An instance of the Single-Row FLP (SRFLP) consists of n one-dimensional departments with given positive lengths $\ell_1, \ldots, \ell_n$ and pairwise costs c_{ij}. The problem is to find a permutation of the departments that minimizes the weighted sum of the pairwise distances. Fig. 1 provides an illustration of the SRFLP in the context of placing the departments along the path of an automated guided vehicle (AGV) transporting material between the departments; in this context the objective is to minimize the distance travelled by the AGV. The SRFLP is the most studied of the row FLPs. Sometimes called the one-dimensional space allocation problem, it has interesting connections to well-known combinatorial optimization problems such as maximum-cut, quadratic linear ordering, and linear arrangement (see Anjos & Liers, 2012).

Because there is only one row, there is no need to assign departments to rows. Moreover, $c_{ij} \geq 0$ ensures that there is no empty space between departments at optimality. Hence the remaining question is to express the center-to-center distance between departments.

A key observation, first made by Simmons (1969), is that the SRFLP can be expressed as

$$\min_{\pi \in \Pi_n} \sum_{i<j} c_{ij} \left[\frac{1}{2} \left(\ell_i + \ell_j \right) + D_\pi (i, j) \right],$$

where Π_n denotes the set of all permutations of $\{1, 2, \ldots, n\}$, and $D_\pi(i, j)$ is the center-to-center distance between departments i and j under permutation π.

A first observation here that if π' denotes the permutation symmetric to π, defined by $\pi'_i = \pi_{n+1-i}, i = 1, \ldots, n$, then $D_\pi (i, j) = D_{\pi'} (i, j)$. In other words, the order of the departments in a particular layout can be reversed without changing the value of the objective function. Hence, it is possible to simplify the problem by considering only the permutations that have a particular