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Discrete Optimization

An algorithmic framework for the exact solution of tree-star problems

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ABSTRACT

Many problems arising in the area of telecommunication ask for solutions with a tree-star topology. This paper proposes a general procedure for finding optimal solutions to a family of these problems. The family includes problems in the literature named as *connected facility location*, *rent-or-buy* and *generalized Steiner tree-star*. We propose a solution framework based on a branch-and-cut algorithm which also relies on sophisticated reduction and heuristic techniques. An important ingredient of this framework is a dual ascent procedure for asymmetric connected facility location. This paper shows how this procedure can be exploited in combination with various mixed integer programming formulations. Using the new framework, many benchmark instances in the literature for which only heuristic results were available so far, can be solved to provable optimality within seconds. To better assess the computational performance of the new approach, we additionally consider larger instances and provide optimal solutions for most of them too.

1. Introduction

Many problems arising in the area of telecommunication ask for solutions possessing a tree-star topology. Consider a scenario in which, for example, servers have to be located on a network that will contain (or cache) information. Information on the servers is updated over time, and this incurs a fixed cost for every edge in the network along which this information is sent. Consequently, the cheapest way to update information over servers (once a choice of which servers to open has been made) is via a *tree* connecting all of them. End-clients requiring the information are served from their closest servers among a *star* topology. This family of tree-star problems includes several known problems in the literature as the *connected facility location*, *rent-or-buy* and *generalized Steiner tree-star problems in graphs*. The problems addressed in this paper are described in the following.

Let I be the set of potential locations where facilities can be opened, and J be the set of customers. Assume that $I \cap J = \emptyset$ and that each customer must be assigned to one open facility. Potential connections between facilities and customers are given by arc set $A_J \subseteq I \times J$. Open facilities must be connected to each other using the so-called *core-network* that consists of I and a set K of intermediate nodes. The set $S = I \cup K$ are called Steiner (or core) nodes. The edge set E_S defines potential connections between Steiner nodes, i.e., the core-network. In the following, we assume that the core-network is connected and that the optimal solution contains at least two facilities. Both assumptions are without loss of generality because, in the first case, we can solve our problem for each component individually and, in the second case, we can consider all single-facility solutions in a preprocessing phase.

The *connected facility location problem (ConFL)* consists of deciding which facilities from I to open, how to connect them using the edges in E_S and how to assign the customers to open facilities. There are costs associated to opening facilities, connecting them through the core network, and assigning customers to facilities. The aim of the ConFLis to find a minimum-cost solution. Fig. 1a shows an instance of the problem and Fig. 1b shows a feasible solution.

In the last decade, the ConFLand variants of it have received much attention from the scientific community, both in the areas of Operations Research and Computer Science (see e.g., (Gollowitzer & Ljubić, 2011; Grandoni & Rothvoß, 2011; Leitner & Raidl, 2011; Swamy & Kumar, 2004) and the references therein). The ConFLhas been independently introduced by Havet and Wennink (2004); Karger and Minkoff (2000); Krick, Räcke, and Westermann (2003) to model applications arising in information/content distribution networks. It has been also shown that the problem is of importance for applications in emergency management

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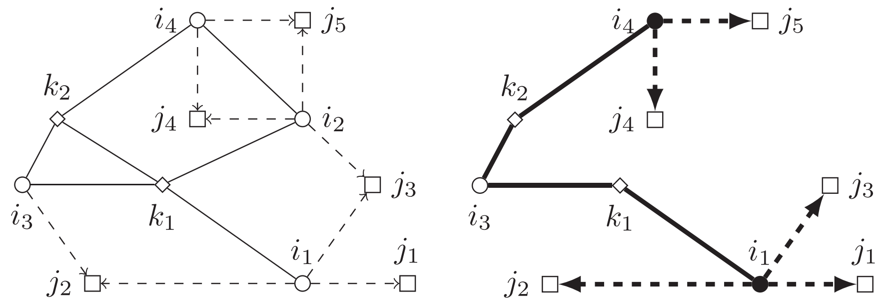

(a) Instance of the ConFL. (b) Feasible solution of the ConFL.

Fig. 1. Example of the ConFLwith $I = \{i_1, \dots, i_4\}$, $J = \{j_1, \dots, j_5\}$ and $K = \{k_1, k_2\}$. The solution is a tree-star structure. Open facilities i_1 and i_4 are indicated in black. Facility i_3 is used for connecting i_1 and i_4 but is not opened.

(Zhu, Huang, Liu, & Han, 2012) where facilities correspond to distribution centers, demand nodes are cities and the facilities are to be connected over a transportation network. Gollowitzer and Ljubić (2011) proposed several mathematical formulations based on direct graphs and compared their linear-programming relaxations. Recently, the problem has been studied from a polyhedral point of view in Leitner, Ljubić, Salazar-González, and Sinnl (2016) (undirected model) and Leitner, Ljubić, Salazar-González, and Sinnl (2014) (rooted directed model). The currently best known approximation ratio for ConFLis 3.19, see (Grandoni & Rothvoß, 2011). A directed version of ConFLis as hard to approximate as the directed Steiner tree problem (also known as the Steiner arborescene problem, see Section 3). Charikar et al. (1999) provide an approximation algorithm for the latter problem with a ratio of $O(k^{\epsilon})$, for any fixed $\epsilon > 0$, where k is the number of nodes that need to be connected.

The *Steiner tree-star problem (STS)* has been introduced by Lu, Qiu, Glover, (1993). In this problem, we are searching for a tree-star network of minimum cost. All customers from J are required to be leaves, and, besides the costs incurred on the edges, non-negative costs need to paid for every node from S which is taken in the solution. Therefore, each node in S is considered as a potential facility, and the facility opening costs have to be paid even if no customer is attached directly to this node. The problem arises in the design of telecommunications networks for digital data service, where nodes from S correspond to locations where digital switching offices with bridging capabilities (called hubs) need to be installed. For the STS, Lee, Chiu, and Ryan (1996) present a branch-and-cut algorithm based on an undirected graph representation, and provide some facet-defining inequalities. A tabu-search heuristic is given in Xu, Chiu, and Glover (1996) and a genetic algorithm in Chu, Premkumar, and Chou (2000).

The *generalized Steiner tree-star problem (GSTS)* is an extension of the STS which has been introduced in Khuller and Zhu (2002), where the authors present approximation algorithms. In this problem, in contrast to the STS, the facility set I and the customer set J do not need to be disjoint, i.e., $I \cap J \neq \emptyset$ is allowed. By introducing a new node j_i for every $i \in I \cap J$ and a zero-cost edge $\{i, j_i\}$, a GSTS instance can be transformed into a STS instance.

A special case of the GSTS has been addressed in Nuggehalli, Srinivasan, and Chiasserini (2003); Swamy and Kumar (2004), under the name of *rent-or-buy problem (ROB)*. In the ROB, every node in the graph can act as facility node, i.e., $J \subseteq I$, and there are no opening costs for facilities. The current best approximation algorithm for the ROB has a factor of 2.92 and is given in Eisenbrand, Grandoni, Rothvoß, and Schäfer (2010).

Bardossy and Raghavan (2010) introduced a variant of the ConFLin which the opening costs have to be paid for each facility, no matter if it serves a customer or not. The ConFL, STS, GSTS and ROB are transformed into this variant in Bardossy and Raghavan (2010), which is solved heuristically using a primal-dual approach in combination with local search. Their transformation involves increasing the number of nodes and edges. Thus, in particular when larger graphs are considered, the performance of their approach is significantly deteriorated.

Our paper proposes a new approach for finding provably optimal solutions for the family of tree-star problems. The approach is based on a branch-and-cut framework using various Mixed-Integer Programming (MIP) formulations. First, we show that all the considered tree-star problems can be seen as special cases of the asymmetric version of the ConFL, denoted by *aConFL*. Then, we exploit a connection between the dual ascent approach for the Steiner arborescence problem (see, e.g., Polzin & Daneshmand, 2001; Wong, 1984) and various cut-based formulations for the aConFL. In addition, the approach contains sophisticated reduction techniques and heuristic procedures. In this article we also demonstrate how Benders decomposition can be used to project out the "stars" from the underlying models.

We assess the efficiency of our approach with an extensive computational study using benchmark instances previously proposed in Bardossy and Raghavan (2010); Gollowitzer and Ljubić (2011). The implementation solves to optimality all instances from (Bardossy & Raghavan, 2010) for which the optimal solutions were unknown, and it also gives a considerable speed-up on the instances from (Gollowitzer & Ljubić, 2011). For that reason we also introduce a new set of larger benchmark instances, some of which are solved to optimality. Finally, the construction heuristic used in our approach turns out to be competitive in solution quality, and much faster in runtime, when compared to the heuristic approach in Bardossy and Raghavan (2010).

The rest of the paper is organized as follows. Section 2 defines the aConFL problem and shows how to transform each of the above-considered tree-star problems into the aConFL. In addition, the section also gives integer programming formulations for the aConFL. Section 3 points out the connection between the aConFL and the Steiner arborescence problem, and exploits this relation to build an efficient branch-and-cut algorithm. It details a dual ascent algorithm that provides strong cuts for our branch-and-cut algorithm. The section also describes reduction tests based on reduced costs obtained from the dual ascent algorithm, and introduces construction and primal heuristics. Further implementation details of the framework are also given in Section 3. Section 4 analyzes computational results. Section 5 summarizes the paper.

2. Mathematical formulations

In this section, we first define the aConFL and transform the four tree-star problems mentioned above into it. We then recall