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Discrete Optimization

Minimum Spanning Trees with neighborhoods: Mathematical programming formulations and solution methods

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ABSTRACT

This paper studies Minimum Spanning Trees under incomplete information assuming that it is only known that vertices belong to some neighborhoods that are second order cone representable and distances are measured with a ℓ_q-norm. Two Mixed Integer Non Linear mathematical programming formulations are presented, based on alternative representations of subtour elimination constraints. A solution scheme is also proposed, resulting from a reformulation suitable for a Benders-like decomposition, which is embedded within an exact branch-and-cut framework. Furthermore, a mathheuristic is developed, which alternates in solving convex subproblems in different solution spaces, and is able to solve larger instances. The results of extensive computational experiments are reported and analyzed.

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1. Introduction

Nowadays Combinatorial Optimization (CO) lies in the heart of multiple applications in the field of Operations Research. Many such applications can be formulated as optimization problems defined on graphs where some particular structure is sought satisfying some optimality property. Traditionally this type of problems assumed implicitly the exact knowledge of all input elements, and, in particular, of the precise position of vertices and edges. Nevertheless, this assumption does not always hold, as uncertainty, lack of information, or some other factors may affect the relative position of the elements of the input graph. Hence, new tools are required to give adequate answers to these challenges, which have been often ignored by standard CO tools.

A matter that, in this context, has attracted the interest of researchers over the last years is the solution of certain CO problems when the exact position of the vertices of the underlying graph is not known with certainty. If probabilistic information is available, then stochastic programming tools can be used, and optimization over expected values carried out. Moreover, even under the assumption of incomplete information one could use a uniform distribution and still apply such an approach. However, the use of probabilistic information and allowing to consider all possible locations for the vertices is not always suitable. For instance, when a unique representative associated with each point of the input graph must be determined. Scanning the related literature one can find papers applying both methodologies. Examples of stochastic approaches are for instance Bertsimas and Howell (1993) or Frank (1969). Examples of the second type of approach arise in variants of the traveling salesman problem (TSP), Minimum Spanning Tree (MST), or facility location problems that deal with *demand regions* instead of *demand points* (see Arkin and Hassin, 1994; Brimberg and Wesolowsky, 2002; Cooper, 1978; Dror, Efrat, Lubiw, and Mitchell, 2003; Juel, 1981; Nickel, Puerto, and Rodríguez-Chía, 2003; Yang, Lin, Xu, and Xie, 2007, to mention just a few).

A relevant common question raised by the latter class of problems is how to model and solve optimization problems on graphs when vertices are not points but regions in a given domain. The above mentioned case of the TSP, first introduced by Arkin and Hassin (1994, 2000), has been addressed recently by a number of authors. It generalizes the Euclidean TSP and the group Steiner tree problem, and has applications in VLSI-design and other routing problems, in which there exist constraints on the position of the vertices. Several inapproximability results and approximation algorithms have been developed for particular cases. The case of the spanning tree problem with neighborhoods (MSTN) was first addressed by Yang et al. (2007), who proved that the general case of the problem in the plane is NP-hard (result also reproved by Löffler & van Kreveld, 2010), and gave several approximation algorithms and a PTAS for the particular case of disjoint unit disks in the plane. Some extensions considering the maximization of the

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weights are studied in Dorrigiv et al. (2013). In particular, they proved the non existence of FPTAS for MSTN, for general disjoint disks, in the planar Euclidean case. Disser, Mihalák, Montanari, and Widmayer (2014) consider the shortest path problem and the rectilinear MSTN, and give some approximability results. To the best of our knowledge, Gentilini, Margot, and Shimada (2013) are the first authors to propose an exact Mixed Integer Non Linear Programming (MINLP) formulation for the TSP with neighborhoods, but we are not aware of any MINLP for the MSTN.

Our goal in this paper is to develop MINLP formulations and solution methods for the MSTN. We first present two MINLP formulations that allow to solve medium size MSTN planar and 3D Euclidean instances with up to 20 vertices, for neighborhoods of varying radii using an on-the-shelf solver. Furthermore, we develop an effective branch-and-cut strategy, based on a generalized Benders decomposition (Benders, 1962; Geoffrion, 1972), and compare its performance with that of the solver for the proposed formulations. For this we present an alternative formulation for the MSTN, in which the master problem consists of finding a MST with costs derived from a continuous non linear (slave) subproblem, and we develop the expression and separation of the cuts that are added in the solution algorithm. Given that both the solver (for the two MINLP formulations) and the exact branch-and-cut algorithm can be too demanding, in terms of their computing times, we have also developed an effective and efficient mathheuristic. The mathheuristic stems from the observation that the subproblems defined in the solution spaces of each of the two main sets of variables are convex (so they can be solved very efficiently); it alternates in solving subproblems in each of these solution spaces.

The paper is organized as follows. Section 2 is devoted to introduce the MSTN and to state a generic formulation. In Section 3 we present and compare two MINLP formulations for the MSTN, based on alternative representations of the spanning trees polytope. Section 4 develops the exact branch-and-cut algorithm, based on a Benders-like decomposition scheme: we define the master and the non linear subproblem, and derive the cuts and their separation. In Section 4.1 we first compare the performance of the on-the-shelf solver with the two MINLP formulations, and then we report the numerical results obtained with the exact row-generation algorithm. The mathheuristic is presented in Section 5, where we also give the numerical results that it produces. The paper ends with some concluding remarks and our list of references.

2. Minimum Spanning Trees with neighborhoods

Let $G=(V,E)$ be a connected undirected graph, whose vertices are embedded in $\mathbb{R}^d$, i.e., $v\in\mathbb{R}^d$ for all $v\in V$. Associated with each vertex $v\in V$, let $\mathcal{N}_v\subseteq\mathbb{R}^d$ denote a convex set containing v in its interior. Let also $\|\cdot\|$ denote a given norm.

Feasible solutions to the Minimum Spanning Tree with Neighborhoods (MSTN) problem consist of a set of points, $Y^*=\{y_v\in\mathcal{N}_v \mid v\in V\}$, together with a spanning tree T^* on the graph $G^*=(Y^*,E^*)$, with edge set $E^*=\{\{y_v,y_w\}:\{v,w\}\in E\}$. Edge lengths are given by the norm-based distance between the selected points relative to $\|\cdot\|$, i.e.:

$$d(y_v,y_w)=\|y_v-y_w\|,\qquad \text{for all } \{y_v,y_w\}\in E^*.$$

The overall cost of (Y^*, T^*) is therefore

$$d(T^*)=\sum_{e=\{y_v,y_w\}\in T^*} d(y_v,y_w).$$

The MSTN is to find a feasible solution, (Y^*, T^*), of minimum total cost.

Particular cases of the MSTN have been studied in the literature for planar graphs. Disser et al. (2014) studied the case when the sets $\mathcal{N}_v$ are rectilinear neighborhoods centered at $v\in V$. Dorrigiv et al. (2013) addressed the problem when the sets $\mathcal{N}_v$ are disjoint Euclidean disks. Both referenced works study the complexity of the considered problems but do not attempt to develop MINLP formulations or solution methods for it.

In this paper, we consider the general case where the graph G is embedded in $\mathbb{R}^d$. Even if our developments can be extended to generic convex sets, we focus on the case where $\mathcal{N}_v$ is second order cone (SOC) representable (Lobo, Vandenberghe, Boyd, & Lebret, 1998). The main reason for this is that state-of-the-art solvers incorporate mixed integer non-linear implementations of SOC constraints. Such a modeling assumption could be readily overcome if on-the-shelf solvers incorporated more general tools to deal with convex sets.

Observe that SOC representable neighborhoods allow to model, as a particular case, centered balls of a given radius r_v, associated with the standard ℓ_p-norm with $p\in[1,\infty]$ in $\mathbb{R}^d$, that we denote by $\|\cdot\|_p$, i.e., neighborhoods in the form $\mathcal{N}_v=\{x\in\mathbb{R}^d:\|x-v\|_p\le r_v\}$, where

$$\|z\|_p=\begin{cases}\left(\sum_{k=1}^d |z_k|^p\right)^{\frac{1}{p}} & \text{if } p<\infty\\ \max_{k\in\{1,\dots,d\}}|z_k| & \text{if } p=\infty\end{cases}.$$

The reader is referred to Blanco, Puerto, and El-Haj Ben-Ali (2014) for further details on the SOC constraints that allow to represent (as intersections of second order cone and/or rotated second order cone constraints) such norm-based neighborhoods. Indeed, we can also easily handle neighborhoods defined as bounded polyhedra in $\mathbb{R}^d$, as well as intersections of polyhedra and balls. Hence, more sophisticated convex neighborhoods can be suitably represented or approximated using elements from the above mentioned families of sets.

Two extreme situations that can be modeled within our framework are the following. If the neighborhood for each vertex $v\in V$ is the singleton $\mathcal{N}_v=\{v\}$, then MSTN becomes the classical MST problem with edge lengths given by the norm-based distances between each pair of vertices. On the other hand, if the considered neighborhoods are big enough so that $\bigcap_{v\in V}\mathcal{N}_v\neq\emptyset$, then the problem reduces to finding a degenerate one-vertex tree and the solution to the MSTN is that vertex with cost 0.

Throughout this paper we use the following notation:

- $\mathcal{ST}_G$ as the set of incidence vectors associated with spanning trees on G, i.e. $\mathcal{ST}_G=\{x\in\mathbb{R}_+^{|E|}: x \text{ is a spanning tree on } G\}$.
- $\mathcal{Y}=\prod_{v\in V}\mathcal{N}_v$, where $\mathcal{N}_v$ is the neighborhood associated to vertex v, which contains the possible sets of vertices for the spanning trees of MSTN.

Then, the MSTN can be stated as:

$$\min \sum_{e\in E} d(y_v,y_w)x_e \qquad \text{(MSTN)}$$
$$\text{s.t.}\quad x\in\mathcal{ST}_G,\ y\in\mathcal{Y}.$$

Several observations follow from the formulation above:

1. Fixing $x\in\mathcal{ST}_G$ in MSTN results in a continuous SOC problem, which is well-known to be convex (Lobo et al., 1998). On the other hand, fixing $y\in\mathcal{Y}$ results in a standard MST problem. It is a well-known that MST admits continuous linear programming representations (Edmonds, 1970; Martin, 1991). Thus, MSTN can be seen as a biconvex optimization problem, which is neither convex nor concave (Gorski, Pfeuffer, & Klamroth, 2007).
2. Due to the expression of its objective function, (MSTN) is not separable, even if each of its sets of variables x and y belong to convex domains in different spaces.
3. Since (MSTN) *combines* the above two subproblems, it is suitable to be represented as a MINLP.

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