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# The continuous single source location problem with capacity and zone-dependent fixed cost: Models and solution approaches☆

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ABSTRACT

The continuous capacitated single-source multi-facility Weber problem with the presence of facility fixed cost is investigated. A new mathematical model which incorporates multi-level type capacity (or design) and facility fixed cost that is capacity-based and zone-dependent is introduced. As no data set exists for this new location problem, a new data set based on convex polygons using triangular shape is constructed. A generalised two stage heuristic scheme that combines the concept of aggregation, an exact method, and an enhanced Cooper's alternate location–allocation method is put forward. A framework that embeds Variable Neighbourhood Search is also proposed. Computational experiments show that these matheuristics produce encouraging results for this class of location problems. The proposed approaches are also easily adapted to cater for a recently studied variant namely the single-source capacitated multi-facility Weber problem where they outperform those recently published solution methods.



## 1. Introduction

The Multi-facility Weber problem (MFWP) deals with finding the location of $m$ facilities in the continuous space and the allocation of each customer to the $m$ chosen facilities so that the sum of the total transportation costs is minimised. This problem, also known as the planar location–allocation problem, is classified as the Multi-facility Weber problem if the demand or weight of all customers is unity and as the generalised MFWP otherwise. Cooper (1963 and 1972) shows that the objective function of MFWP is neither concave nor convex and may contain multiple local minima which makes the problem difficult to solve using exact methods. Hence, the MFWP falls in the realm of global optimisation problems. In addition, the MFWP is shown to be NP-hard, see Megiddo and Supowit (1984) and Sherali and Nordai (1988).

The single source location problem arises in situations where customers must be served by one facility only which is referred to as the single source capacitated multi-facility Weber problem (SCCMFWP). For instance, in a telecommunication network design, a user is assigned to a single base transceiver station, while when locating oil drill platforms, each oil well has to be allocated to one platform (Devine & Lesso, 1972 and Rosing, 1992). In real life applications, it is also worth taking into account a set up or opening cost of a facility which may be dependent on geographical areas (zones) and/or a throughput rate (capacity) of the facility. For example, for political, environmental or economic reasons, there are some governments that implement different tax policies for urban, suburban, and remote regions or regional restrictions as some areas are under government protections such as forests, lakes, rivers, etc. As a result, in some areas there may be cheaper opening costs of locating a facility whereas in others extortionate costs could be imposed. This paper proposes a mathematical model and solution methods to deal with such a strategic decision problem which, in many cases, require a massive investment.

In this study we aim to

(i) propose a new mathematical model for the SCCMFWP considering the fixed costs that are capacity-based and zone-dependent (SSCMFWP-FC)
(ii) develop an effective two stage heuristic approach and introduce an enhanced Variable Neighbourhood Search
(iii) construct new data sets for the SSCMFWP-FC and record promising results, and
(iv) produce several new best solutions for the recently studied location problem variant namely the SSCMFWP.

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The paper is organised as follows. In the next section, a review of the relevant literature is provided. The section thereafter presents the mathematical models for both the SSCMFWP and the SSCMFWP-FC. In Section 4, the proposed solution frameworks are described followed by the computational results in Section 5. The adaptation and the implementation of our approach for the SSCMFWP problem are presented in Section 6. Finally, our conclusions and some highlights of future research are given in the last section.

## 2. Literature review

In this section we first briefly look at works that treat problems similar to ours. This is followed by a more detailed description of the few papers on the single-source capacitated multi-facility Weber Problem itself. For a comprehensive review on the MFWP which has attracted much research attention in the literature, the reader can refer to the works of Brimberg, Hansen, Mladenović, and Salhi (2008) and Brimberg, Drezner, Mladenović, and Salhi (2014). The discrete case is not reviewed here, but for completeness see Correia and Captivo (2003) for the capacitated case, and Correia and Captivo (2006) for the single source case.

### *2.1. A brief on the capacitated MFWP (CMFWP)*

Cooper (1972) is among the first who puts forward exact and heuristic methods for the CMFWP. The latter is the well-known alternating transportation-location (ATL for short) heuristic. Basically, ATL is a modification of the heuristic (ALA) originally developed by Cooper (1964) for the MFWP. This technique is based on alternately solving the location–allocation problem and the Transportation Problem (TP) until there is less than epsilon ($\varepsilon$) improvement found in the total cost. It is worth noting that once the facilities are located, the CMFWP reduces to the classical TP. Sherali and Shetty (1977) propose a convergent cutting plane algorithm, which is originally derived from a bilinear programming problem, to solve the rectilinear distance CMFWP.

Sherali and Tuncbilek (1992) revisit the problem using a distance proportional to the square of the Euclidean distance. They design a branch and bound algorithm to compute strong upper bounds. Sherali, Ramachandran, and Kim (1994) formulate the rectilinear distance CMFWP as a mixed integer nonlinear programming model, and develop a reformulation-linearization technique to transform the problem into a linear mixed-integer program. Sherali, Al-Loughani, and Subramanian (2002) also design a branch and bound technique based on partitioning the allocation space to construct global optimisation procedures for the Euclidean and $\mathbf{l}_p$ distance CMFWP.

Zainuddin and Salhi (2007) propose a perturbation-based heuristic to tackle the CMFWP. This scheme considers borderline customers whose locations lie approximately half-way between their nearest and their second nearest facilities. Aras, Altınel, and Orbay (2007a) develop three heuristic techniques which include Lagrangean heuristic, the discrete $p$-capacitated facility location heuristic which is similar to the $p$-median method of Hansen, Mladenović, and Taillard (1998), and the cellular heuristic of Gamal and Salhi (2003) to solve the CMFWP with Euclidean, squared Euclidean, and $l_p$ distances. Aras, Yumusak, and Altınel (2007b) adopt simulated annealing, threshold accepting, and genetic algorithms to deal with the CMFWP with rectilinear, Euclidean, squared Euclidean, and $l_p$ distances. In a following study, Aras, Orbay, and Altinel (2008) adapt their earlier approaches to tackle the CMFWP with rectilinear distance.

Luis, Salhi, and Nagy (2009) study the CMFWP by designing restricted regions within their constructive heuristic which forbid new locations to be sited too close to the previously found locations. A discretisation method which divides a continuous space into a discrete number of cells while embedding the use of restricted regions within the search is also put forward. Mohammadi, Malek M, and Alesheikh (2010) design two genetic algorithms (GAs); one for the location problem and the other for the allocation of customers to those open facilities. Luis, Salhi, and Nagy (2011) present a novel guided reactive greedy randomised adaptive search procedure by designing a framework that combines adaptive learning with the concept of restricted regions. Akyüz, Altınel, and Öncan (2014) study the CMFWP by developing two branch and bound algorithms with the first designed for the allocation space whereas the second for the partition of the location space.

### *2.2. A brief on the SSCMFWP*

The literature on the SSCMFWP is very scarce. Gong, Gen, Yamazaki, and Xu (1997) propose a hybrid evolutionary method based on GA to search the locatable area and hence find the global or near global solutions. In the allocation stage, a Lagrangean relaxation approach is applied. Experiments are carried out on randomly generated data with the number of facilities ($m$) varying from 2 to 6.

Manzour-al-Ajdad, Torabi, and Eshghi (2012) develop an iterative two phase heuristic algorithm to tackle the problem. In the first phase or location phase, the ALA method of Cooper (1964) is modified by introducing two assignment rules namely the simplified and parallel assignments, respectively. In the second phase or the allocation phase, customers are allocated to facilities by solving optimally the generalised assignment problem. A simulated annealing algorithm is also used as an alternative solution in the first phase. Data sets taken from the literature are adapted accordingly to cater for the SSCMFWP. The results are compared with a general MINLP solver BARON that is run for a limited time. The authors claim that their proposed methods provide better results than BARON. Manzour, Torabi, and Pishvaee (2013) produce a simpler version to the one proposed by Manzour-al-Ajdad et al. (2012) but with slightly inferior results.

Öncan (2013) investigates the SSCMFWP with Euclidean and Rectilinear distances by proposing three solution methods. The first one is the Single-Source ALA method which is an improved version of Cooper's ALA method (Cooper, 1964) when the allocation phase is solved optimally. In the second one, a very large neighbourhood search procedure is employed within the first method to solve the allocation problem efficiently. In the third method, a discrete approximation technique that uses Lagrangean Relaxation is put forward to find lower and upper bounding procedures for the SSCMFWP. Experiments are performed using three classes of instances from the literature as well as newly randomly generated data sets. Competitive empirical results are produced when compared to the published work though these are found to be relatively inferior in some instances to those given by Manzour-al-Ajdad et al. (2012).

### *2.3. A brief on the Weber problem in the presence of fixed cost*

Most of the literature on the facility location problems with fixed costs focus on the discrete space, see for instance the recent papers by Farahani, Masoud Hekmatfar, Fahimnia, and Kazemzadeh (2014), Guastaroba and Speranza (2014), Rahmani and MirHassani (2014), and Ho (2015). However, in the continuous location problem there is a shortage of references which investigate the presence of fixed costs. Brimberg, Mladenović, and Salhi (2004) study the multi-source Weber problem with constant fixed cost and design a multiphase heuristic to deal with the problem. The discrete version of the problem is first solved to find an approximate number of facilities and then the facility configuration is improved by applying Cooper's ALA scheme. Brimberg and Salhi (2005) propose fixed costs which are zone-dependent for locating