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Innovative Applications of O.R.

Curvature-constrained traveling salesman tours for aerial surveillance in scenarios with obstacles

Luitpold Babel

Institut für Mathematik und Informatik, Fakultät Betriebswirtschaft, Universität der Bundeswehr München, Neubiberg D-85579, Germany

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ABSTRACT

The curvature-constrained traveling salesman problem with obstacles deals with finding a minimum length tour which includes a set of landmarks and avoids obstacles, for a kinematically constrained vehicle. Its great practical importance is mainly due to surveillance tasks of unmanned aerial vehicles. The problem constitutes a combination of the well-studied Dubins traveling salesman problem and the flight path planning problem. We present heuristic algorithms that are based on different strategies of extending a tour by inserting new landmarks. Each insert operation comprises the optimization of overflight directions for the given sequence of landmarks. Path finding between landmarks is done by a discrete routing model. It allows arbitrary flight directions and turn angles as well as maneuvers of different strengths, thus fully exploiting the flight capabilities of the aircraft. The performance of the algorithms is evaluated for agile and less agile aerial vehicles, using randomly generated scenarios with obstacles of different size and number.

1. Introduction

Unmanned aerial vehicles (UAVs) play an important role in providing high quality surveillance information, both in civilian and military missions. These operations require visiting multiple targets and capturing images by onboard cameras. An evaluation of the recorded data is done either on completion of the mission, or online by transmitting the information via data link to a remote operating station. The range of application comprises

- traffic control over specific locations
- intelligence gathering and reconnaissance of suspicious targets for anti-terrorism operations
- security missions and monitoring of critical infrastructure and other points of interests
- support of combat missions by intelligence, surveillance and reconnaissance (ISR) operations
- battle damage assessment (i.e. confirming a target and verifying its destruction) and others.

UAV missions need to be cost-efficient in terms of flight time or length of the flight route. In this context, route planning is a major challenge, in particular since obstacles in the operational area may significantly complicate the task. Such obstacles include

- topographic elevations (hills and mountains)
- tall structures (buildings, power lines, broadcasting towers, antenna installations)
- threats to the integrity of the UAV (air defense, hostile radar sites, countermeasures)
- nofly areas (neutral zones, populated areas, locations of friendly forces)

and others. Route planning involves the search for a collision-free path, taking into account the flight capabilities of the aerial vehicle.

In this paper, we consider the task of finding a minimum length tour in the horizontal plane from a release point to a destination point which includes a set of landmarks (targets, points of interests, etc.) and avoids obstacles. Release and destination point may be identical or different. In the first case the tour is closed. Both points have predefined release and approach directions. For a closed tour, the directions are often identical or have an offset of 180° (take-off and landing on a runway in the same or the opposite direction). One key issue is to determine a suitable order of the landmarks and suitable overflight directions. No less important is to generate a tour that is *feasible*, i.e. the trajectory does not collide with obstacles, and *flyable*, i.e. the trajectory respects the kinematic properties of the aerial vehicle.

In the literature, a great deal of attention is paid to variations of the *Traveling Salesman Problem* (TSP), where a tour of minimum length has to be found that passes through every target location precisely once (see e.g. Gutin & Punnen, 2007; Laporte, 1992; Lawler, Lenstra, Rinnooy Kan, & Shmoys, 1985). The *Euclidean*

E-mail address: luitpold.babel@unibw.de

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Traveling Salesman Problem (ETSP) is a TSP where the distances between the vertices are precisely the Euclidean distances of the target locations in the plane. The *Dubins Traveling Salesman Problem* (DTSP) refers to a kinematically constrained vehicle, also known as *Dubins vehicle*. It moves at a constant speed, cannot reverse, and has a minimum turning radius. Dubins vehicles are widely used for motion planning of walking robots, aircraft, ships, etc. (see LaValle, 2006). Since the curvatures of the associated trajectories are restricted, the DTSP is also known as *bounded-curvature TSP*.

The *Traveling Salesman Problem with Neighborhoods* (TSPN) extends the traveling salesman problem to the case where each vertex of the tour is allowed to move in a given region (see e.g. Arkin & Hassin, 1994). This version of the TSP takes into account the communication range or the sensor footprint of the aircraft. The *Euclidean TSP with neighborhoods* (ETSPN) seeks for a shortest Euclidean path passing through the regions (see e.g. Dumitrescu & Mitchell, 2003). The *TSPN with a Dubins vehicle* (DTSPN) has first been tackled by Obermeyer, Oberlin, and Darbha (2010, 2012) and further investigated by other researchers (see e.g. Isaacs, Klein, & Hespanha, 2011; Xin, Chen, Xu, & Chen, 2014).

The ETSP is known to be NP-hard (see Papadimitriou, 1977), so are the TSPN and the ETSPN by virtue of being generalizations of the ETSP. Only a few years ago it was formally proven by Le Ny, Feron, and Frazzoli (2012) that the DTSP is NP-hard. As the area of the regions goes to zero, the DTSPN reduces to the DTSP, hence the DTSPN is also NP-hard.

Dubins (1957) investigated the following problem. Given two points in the plane with prescribed initial and terminal tangents, find a shortest path with a constraint on the curvature and with continuous tangent direction. He was able to prove that each such path consists of a concatenation of straight lines and circle segments with maximal curvature. More precisely, each path is of the form CSC or CCC where C stands for a concave or convex circle segment with maximal curvature and S for a line segment. The solutions are commonly called *Dubins paths*. They are used as an ingredient in most algorithms for the DTSP.

Since the DTSP is NP-hard, it seems hardly possible to find optimal solutions for large instances of the problem. Numerous heuristic methods have been developed, many of them with worst-case performance bounds. They fall broadly into three categories:

(A) First determine a suitable order of the landmarks, then find suitable overflight directions.
(B) Find suitable overflight directions, then optimize the order of the landmarks.
(C) Methods that do not separate order and overflight directions (mixed strategies).

Approaches of type (A) have been developed e.g. by Ma and Castanon (2006); Rathinam, Sengupta, and Darbha (2007); Savla, Frazzoli, and Bullo (2008), Macharet and Campos (2014). The algorithms start with a solution of the Euclidean TSP and apply different strategies to determine overflight directions. Representatives of type (B) are the algorithms of Tang and Özgüner (2005) and Le Ny, Frazzoli, and Feron (2007). While the former determines an order of the landmarks by geometric reasoning, the latter succeeds by solving an asymmetric traveling salesman problem. Type (C) includes an incremental method of Le Ny et al. (2012) that extends a path by adding one new target at a time and optimizing its approach direction, and a second method that chooses a set of possible directions at each target and transforms the problem into a generalized asymmetric TSP. Cohen, Epstein, and Shima (2017) discretize the problem and formulate it as an integer optimization problem.

Let us note that most studies of the DTSP do not consider the existence of obstacles. On the other hand, obstacles are a central issue for the *flight path planning problem* (FPP) where a collision-free shortest path from an initial to a destination point has to be found (see e.g. Goerzen, Kong, & Mettler, 2010; Latombe, 1991). A large variety of techniques have been applied for the FPP including the potential field method (see e.g. Kim & Khosla, 1992), cell decomposition (Lingelbach, 2005), the roadmap method (Kavraki, Svestka, Latombe, & Overmars, 1996), rapidly exploring random trees (LaValle & Kuffner, 2001), mixed integer linear programs (Bellingham, Richards, & How, 2002), and diverse network-based approaches (see e.g. Babel, 2012; Bortoff, 2000; De Filippis, Guglieri, and Quagliotti, 2012; Jun & D'Andrea, 2003). Some, but not all, of these algorithms take into account the flight properties of the aerial vehicle.

A lot of research has also been done in order to find shortest obstacle-avoiding Dubins paths. This includes, among others, the work of Agarwal and Wang (2001); Bicchi, Casalino, and Santilli (1996); Gottlieb and Shima (2015); Yang, Li, and Sun (2013). It is well known (see Hershberger & Suri, 1999) that a shortest obstacle-avoiding Euclidean path in a scenario with polygonal obstacles represented by a total of n vertices can be calculated in time $O(n\log n)$. A Euclidean path consists of straight line segments and does not consider curvature constraints. On the other hand, the same problem with a Dubins vehicle is NP-hard in n as shown by Reif and Wang (1998).

The problem treated in this work constitutes an extension of both the DTSP and the FPP, hereinafter also being referred to as the *curvature-constrained TSP with obstacles* (CTSPO). We are thus facing the challenge of a combination of two notoriously hard problems. This confronts us with additional difficulties.

– While there is always a solution for the DTSP, this is not guaranteed for the CTSPO. Badly located obstacles may block the existence of a tour containing all landmarks. In such a case, the objective should be to find a tour with as many landmarks as possible, i.e. to inspect a largest subset of the landmarks.
– Obstacles will often prevent the existence of a Dubins path between two landmarks. One might relax Dubins paths to more complex and longer paths, also consisting of straight lines and circle segments with maximal curvature (i.e. as for Dubins paths, the vehicle is exposed to the maximal possible lateral acceleration). However, a shortest path in a scenario with obstacles might as well contain curves with smaller lateral acceleration.
– Dubins paths have continuous tangent direction. However, this is not true for the curvature. The transition between a straight line and a circle segment implies a sudden centripetal acceleration to the vehicle, which definitely should be avoided in practice. Hence there is a need for smoother trajectories.

The main contribution of this work is to

(i) provide a collection of heuristic algorithms to solve the CTSPO. As an ingredient of these methods, algorithms are proposed that (ii) find a feasible and flyable trajectory between two landmarks, and (iii) find a feasible and flyable tour through a given sequence of landmarks, both as short as possible. The presentation is supported by a numerical study of the algorithms.

The algorithms for the CTSPO are based on different strategies of extending a tour by inserting new landmarks. Each insert operation comprises the optimization of overflight directions for the given sequence of landmarks. This is achieved by applying algorithm (iii), i.e. discretizing the directions, constructing an auxiliary network that implicitly contains all shortest paths between consecutive landmarks, and calculating a shortest path. Path finding between landmarks is done by algorithm (ii). It depends on a discrete routing model that represents the airspace by a sophisticated network. The model allows arbitrary flight directions and turn angles as well as maneuvers of different strengths, thus fully exploiting the flight capabilities of the aircraft. Moreover, the networks are