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Invited Review

The Benders decomposition algorithm: A literature review

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ABSTRACT

The Benders decomposition algorithm has been successfully applied to a wide range of difficult optimization problems. This paper presents a state-of-the-art survey of this algorithm, emphasizing its use in combinatorial optimization. We discuss the classical algorithm, the impact of the problem formulation on its convergence, and the relationship to other decomposition methods. We introduce a taxonomy of algorithmic enhancements and acceleration strategies based on the main components of the algorithm. The taxonomy provides the framework to synthesize the literature, and to identify shortcomings, trends and potential research directions. We also discuss the use of the Benders Decomposition to develop efficient (meta-)heuristics, describe the limitations of the classical algorithm, and present extensions enabling its application to a broader range of problems.

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1. Introduction

It has been more than five decades since the *Benders Decomposition* (*BD*) algorithm was proposed by Benders (1962), with the main objective of tackling problems with *complicating variables,* which, when temporarily fixed, yield a problem significantly easier to handle. The BD method (also referred to as *variable partitioning*, Zaourar and Malick (2014), and *outer linearization*, Trukhanov, Ntaimo, and Schaefer (2010)) has become one of the most widely used exact algorithms, because it exploits the structure of the problem and decentralizes the overall computational burden. Successful applications are found in many divers fields, including planning and scheduling (Canto, 2008; Hooker, 2007), health care (Luong, 2015), transportation and telecommunications (Costa, 2005), energy and resource management (Cai, McKinney, Lasdon, & Watkins, 2001; Zhang & Ponnambalam, 2006), and chemical process design (Zhu & Kuno, 2003), as illustrated in Table 1.

The BD method is based on a sequence of projection, outer linearization, and relaxation (Geoffrion, 1970a, 1970b). The model is first projected onto the subspace defined by the set of complicating variables. The resulting formulation is then dualized, and the associated extreme rays and points respectively define the feasibility requirements (feasibility cuts) and the projected costs (optimality cuts) of the complicating variables. Thus, an equivalent formulation can be built by enumerating all the extreme points and rays. However, performing this enumeration and, then, solving the resulting formulation is generally computationally exhausting, if not impossible. Hence, one solves the equivalent model by applying a relaxation strategy to the feasibility and optimality cuts, yielding a *Master Problem* (*MP*) and a subproblem, which are iteratively solved to respectively guide the search process and generate the violated cuts.

The BD algorithm was initially proposed for a class of mixed-integer linear programming (*MILP*) problems. When the integer variables are fixed, the resulting problem is a continuous linear program (*LP*) for which we can use standard duality theory to develop cuts. Many extensions have since been developed to apply the algorithm to a broader range of problems (e.g., Geoffrion, 1972; Hooker & Ottosson, 2003). Other developments were proposed to increase the algorithm's efficiency on certain optimization classes (e.g., Costa, Cordeau, Gendron, & Laporte, 2012; Crainic, Hewitt, & Rei, 2014). In addition, BD often provides a basis for the design of effective heuristics for problems that would otherwise be intractable (Côté & Laughton, 1984; Raidl, 2015). The BD approach has thus become widely used for linear, nonlinear, integer, stochastic, multi-stage, bilevel, and other optimization problems, as illustrated in Table 2.

Fig. 1 depicts the increasing interest in the BD algorithm over the years. Despite this level of interest, there has been no comprehensive survey of the method in terms of its numerical and the-

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Table 1
Some applications of the Benders decomposition method.

	Reference	Application		Reference	Application
1	Behnamian (2014)	Production planning	17	Jiang et al. (2009)	Distribution planning
2	Adulyasak et al. (2015)	Production routing	18	Wheatley et al. (2015)	Inventory control
3	Boland et al. (2016)	Facility location	19	Laporte, Louveaux, and Mercure (1994)	Traveling salesman
4	Boschetti and Maniezzo (2009)	Project scheduling	20	Luong (2015)	Healthcare planning
5	Botton et al. (2013)	Survivable network design	21	Maravelias and Grossmann (2004)	Chemical process design
6	Cai et al. (2001)	Water resource management	22	Moreno-Centeno and Karp (2013)	Implicit hitting sets
7	Canto (2008)	Maintenance scheduling	23	Oliveira et al. (2014)	Investment planning
8	Codato and Fischetti (2006)	Map labeling	24	Osman and Baki (2014)	Transfer line balancing
9	Cordeau et al. (2006)	Logistics network design	25	Pérez-Galarce et al. (2014)	Spanning tree
10	Cordeau et al. (2001a)	Locomotive assignment	26	Pishvaee et al. (2014)	Supply chain network design
11	Cordeau et al. (2001b)	Airline scheduling	27	Rubiales et al. (2013)	Hydrothermal coordination
12	Corréa et al. (2007)	Vehicle routing	28	Saharidis et al. (2011)	Refinery system network planning
13	Côté et al. (2014)	Strip packing	29	Sen et al. (2015)	Segment allocation
14	Fortz and Poss (2009)	Network design	30	Bloom (1983)	Capacity expansion
15	Gelareh et al. (2015)	Transportation	31	Wang et al. (2016)	Optimal power flow
16	Jenabi et al. (2015)	Power management	32	Errico, Crainic, Malucelli, and Nonato (2016)	Public transit

Table 2
Examples of optimization problems handled via Benders method.

	Reference	Model		Reference	Model
1	Adulyasak et al. (2015)	Multi-period stochastic problem	16	Jenabi et al. (2015)	Piecewise linear mixed-integer problem
2	Behnamian (2014)	Multi-objective MILP	17	Wolf (2014)	Multi-stage stochastic program
3	Cai et al. (2001)	Multi-objective nonconvex nonlinear problem	18	Laporte et al. (1994)	Probabilistic integer formulation
5	Cordeau et al. (2001b)	Pure 0–1 formulation	19	Li (2013)	Large-scale nonconvex MINLP
6	Corréa et al. (2007)	Binary problem with logical expressions	20	Moreno-Centeno and Karp (2013)	Problem with constraints unknown in advance
7	Gabrel, Knippel, and Minoux (1999)	Step increasing cost	21	Bloom (1983)	Nonlinear multi-period problem with reliability constraint
8	Côté et al. (2014)	MILP with logical constraints	22	Osman and Baki (2014)	Nonlinear integer formulation
9	de Camargo et al. (2011)	Mixed-integer nonlinear program (MINLP)	23	Pérez-Galarce et al. (2014)	Minmax regret problem
10	Emami et al. (2016)	Robust optimization problem	24	Pishvaee et al. (2014)	Multi-objective possibilistic programming model
11	Fontaine and Minner (2014)	Bi-level problem with bilinear constraints	25	Raidl et al. (2014)	Integer, bilevel, capacitated problem
12	Fortz and Poss (2009)	Multi-layer capacitated network problem	27	Rubiales et al. (2013)	Quadratic MILP master problem and nonlinear subproblem
13	Gendron et al. (2014)	Binary problem with nonlinear constraints	28	Sahinidis and Grossmann (1991)	MINLP and nonconvex problems
14	Grothey et al. (1999)	Convex nonlinear problem	29	Harjunkoski and Grossmann (2001)	Multi-stage problem with logical and big-M constraints
15	O'Kelly et al. (2014)	MINLP with concave objective function and staircase constraint matrix structure			

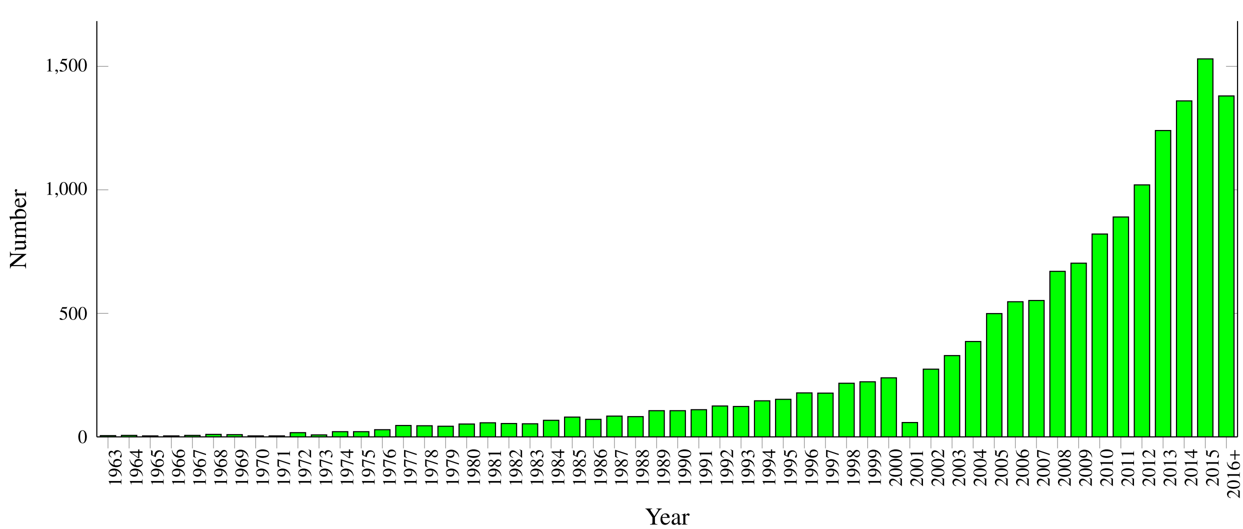

Fig. 1. Annual number of mentions of the Benders decomposition according to https://scholar.google.com/.

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