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Analytical solution for an investment problem under uncertainties with shocks

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A R T I C L E I N F O

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A B S T R A C T

We derive the optimal investment decision in a project where both demand and investment cost are stochastic processes, eventually subject to shocks. We extend the approach used in Dixit and Pindyck (1994) to deal with two sources of uncertainty and we assume that the underlying processes are jump diffusion processes. Assuming certain conditions on the parameters, we are able to derive a closed expression for the value of the firm. Finally, we present comparative statics for the investment threshold with respect to the relevant parameters.

1. Introduction

In this paper we use the real options framework to study an investment decision when both demand and investment cost are random, evolving according to jump diffusion processes.

A real option means that a firm has the right, but not the obligation, to undertake certain business initiatives such as deferring, abandoning, expanding, staging or contracting a capital investment project. The analysis of real options assumes that there are three driving investment factors underpinning the investment decision. The first is that future rewards are uncertain. The second is that the decision is irreversible in the sense that the investment expenditure cannot be fully recovered. The third and last one is that the timing of the investment is variable and therefore the investor may decide on the best time to invest in order to maximize the value of his firm. We refer to Dixit and Pindyck (1994), Arrow and Fisher (1974), McDonald and Siegel (1986), Dixit (1989) and references therein contained as good examples of seminal works on real options.

One of the early works in this area is McDonald and Siegel (1986) wherein the authors model the investment problem when the value of the investment project evolves following a geometric Brownian motion (GBM, for short), and the investor must decide when he should exercise his investment option. The optimal strategy is to exercise the option as soon as the value of the project exceeds a threshold.

In this paper we assume that there are two sources of uncertainty: demand (reflected in the revenue of the firm), and investment cost. Thus the investor needs to optimize his investment decision by taking into account the random fluctuations of the revenue and the changing investment cost.

Problems with two sources of uncertainty are not new, and some examples can be found in the literature, e.g. in the pioneer book of Dixit and Pindyck (1994) and recently Murto (2007), Zambujal-Oliveira and Duque (2011), Pennings and Sereno (2011) and Alghalith (2016).

One of the most common assumption is that the underlying stochastic processes modeling uncertainty are continuous sample-path processes, such as the GBM. This is specially due to its comprehensive analytical properties, as referred by Eberlein (2010). But as this author mentions, *as a consequence of the distributional deficiencies of the classical models that are driven by Brownian motions, advanced models in finance are now based on processes that include jumps or that consist entirely of jumps*. Empirical and theoretical studies already showed the existence of jumps, from portfolio risk management to option and bond pricing (e.g. Hagspiel, Huisman, & Nunes, 2015; Johannes, 2004; Lee & Mykland, 2008; Liu, Longstaff, & Pan, 2003; Merton, 1976; Pan, 2002, among others).

Nowadays, the world global market encompasses exogenous events that may lead to sudden increase or decrease (shocks) in demand for certain products. Clearly, these shocks will considerably affect investors decisions. Taking into account that the

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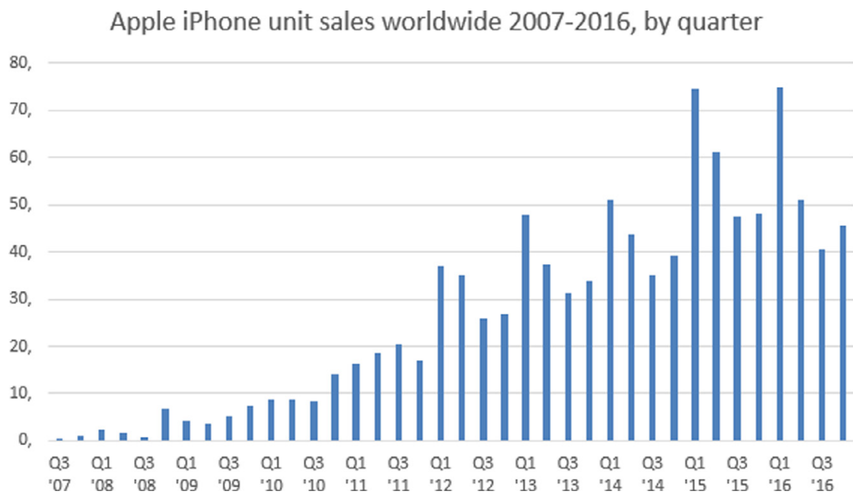

Fig. 1. Global Apple iPhone unit sales from 3rd quarter 2007 to 3rd quarter 2016 (in million units)

involved processes may exhibit sample-path discontinuities, we consider jump-diffusion processes. Several examples support this assumption.

The chart presented in Fig. 1 shows the number of worldwide sales of iPhones from 3rd quarter 2007 to 3rd quarter 2016, with data aggregated by quarters.[1] It is clear that in this case upward jumps occur with a certain frequency, possibly related with the launching of new versions of the iPhone.

Another recent example occurred just after the 9–11 terrorist attack in the USA. One of the headlines of the World Street Journal was *As demand for train service jumps, Amtrak seeks emergency funding*. According to this journal, the Amtrak ridership jumped by 60%.[2]

In the previous two examples, events led to an upward jump on demand. But there are also examples pointing in the opposite direction. The following illustrates a case where a downward jump occurred on demand. In 2015 the Volkswagen's sales decreased, due to the Emissions scandal spread in the news. In fact, reports from Volkswagen announced that global sales fell 2% in 2015.[3] This also had an impact on the price of the company's shares. According to the Wall Street Journal, *shares in Volkswagen AG and Porsche Automobil SE, ..., tumbled by more than 8% in early trade Wednesday ,...*.[4]

We notice that after these jumps sale's level tends to return to previous values, meaning that the impact of the jumps has short memory. This observation is in line with Pimentel, Pereira, and Couto (2012) and Couto, Nunes, and Pimentel (2015), where it is assumed that the demand for high speed rail services can be modeled by a jump-diffusion process. In fact, our work is closely related to these two contributions but considers that demand and investment cost are random.

In the current work we are also interested in changing investment cost, like in Oh, Rhodes, and Strong (2016) or Shi (2016). If one assumes the investment cost is constant and known, then the model is being simplified. This problem is even more important when one considers large investments that take several years to be completed (e.g. construction of dams, nuclear power plants or high speed railways).

Examples of changing investment cost can be found in different areas. One of such examples is related to investment in the energy sector. In Hunt and Shuttleworth (1996) we can find an example where the investment cost decreases: *as a result of studies sponsored by space programs, it was possible to build turbines much more efficient and smaller than before, reducing in a drastic way the optimal power plant size, with enormous cost reduction*.

On the other hand, the investment may also suffer a cost escalation. For example, a report from Mark Cooper, entitled "Policy Challenges of Nuclear Reactor Construction, Cost Escalation and Crowding out Alternatives"[5] refers that ... *the increasing complexity of nuclear reactors and the site-specific nature of deployment make standardization difficult, so cost reductions have not been achieved and are not likely in the future. More recent, more complex technologies are more costly to construct.*

Another example, also taken from the energy sector, concerns the increasing cost to build new transmission infrastructures. In a report from the USA Energy Information Administration, we find the following statement: *the global economic boom of 2004-07, along with a weakened U.S. dollar, raised the prices of raw materials (such as steel and cement), fuel, and labor faster than the rate of inflation*.[6]

Furthermore, besides considering stochastic investment cost, we also allow it to exhibit jumps. Jumps may represent uncertainties arising from the disclosure of new information regarding technological innovation, competition, political risk, regulatory effects and other sources and its impact (Martzoukos & Trigeorgis, 2002; Wu & Yen, 2007).

One paper that is also closely related with our work is Murto (2007), where the author characterizes the timing of investment considering two sources of uncertainty. The author assumes that the revenue is a diffusion process and that the technological uncertainty is a pure Poisson process, which influences the investment cost. In our case, we allow both uncertainty factors to be jump-diffusion processes, combining the continuous behavior (associated with the diffusion) with the discrete behavior (associated with the jumps). Moreover, Murto (2007) presents analytical solutions only for some particular cases, for instance, assuming that one of the

[1] Source: Statista. Available at https://www.statista.com/statistics/263401/global-apple-iphone-sales-since-3rd-quarter-2007/.

[2] Source: The Wall Street Journal, news published on the 21st September 2001. Available at http://www.wsj.com/articles/SB1001021248552406720.

[3] Source: The Guardian, news published on the 8th January. Available at https://www.theguardian.com/business/2016/jan/08/vw-global-sales-fell-year-emissions-scandal-2015.

[4] Source: The Wall Street Journal, news published on the 4th November 2015. Available at http://www.wsj.com/articles/volkswagen-drags-other-auto-shares-lower-on-fears-scandal-could-widen-1446631327.

[5] Available at http://www.psr.org/nuclear-bailout/resources/policy-challenges-of-nuclear.pdf.

[6] Available at http://www.eia.gov/todayinenergy/detail.php?id=17711.