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Discrete Optimization

Effective metaheuristic algorithms for the minimum differential dispersion problem

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ABSTRACT

This paper presents tabu search and memetic search algorithms for solving the minimum differential dispersion problem. The tabu search algorithm employs a neighborhood decomposition candidate list strategy and a rarely used solution-based tabu memory. Unlike the typical attribute-based tabu list, the solution-based tabu strategy leads to a more highly focused intensification process and avoids tuning the tabu tenure, while employing coordinated hash functions that accelerate the determination of tabu status. The memetic search algorithm incorporates the tabu search procedure within it and makes use of a crossover operator that generates solution assignments by an evaluation mechanism that includes both quality and distance criteria. Experimental results on a benchmark testbed of 250 problems reveal that our tabu search algorithm is capable of discovering better solutions for 179 (71.6%) of the problem instances, while our memetic search algorithm finds better solutions for 157 (62.8%) of the instances, collectively yielding better solutions for 181 (72.4%) of the test problems than recently reported state-of-the-art algorithms.

1. Introduction

Let a set $N = \{1, \ldots, n\}$ and a distance d_{ij} ($d_{ij} \in R, d_{ij} = d_{ji}$) between any two elements $i, j \in N$. The equitable dispersion problem consists in selecting a subset $M \subseteq N$ of cardinality m (hence $|M| = m$), to optimize a given equity measure (function) $z(M)$ of d_{ij}. Prokopyev, Kong, and Martinez-Torres (2009) proposed several equity measures that define different dispersion problems, including: Max-Mean DP that maximizes the mean dispersion of the selected elements, Generalized Max-Mean DP that extends the mean dispersion by weighting the elements, Min-DiffSum DP that minimizes the difference between the maximum aggregate dispersion and minimum aggregate dispersion, as well as Max-MinSum DP that maximizes the minimum aggregate dispersion among selected elements. In addition, Punnen, Taghipour, Karapetyan, and Bhattacharyya (2014) proposed a balanced quadratic optimization problem that minimizes the difference between the maximum dispersion and the minimum dispersion among selected elements.

Equitable dispersion problems are a class of NP-hard combinatorial optimization problems (Prokopyev et al., 2009) and have applications in the selection of homogeneous groups (Brown, 1979a), equity based measures in network flows (Brown, 1979b), dense subgraph identification (Kortsarz & Peleg, 1993), urban public facility location (Teitz, 1968) and tour package planning (Punnen et al., 2014). In particular, equity dispersion problems include applications that are closely related to public facility locations (Barbati & Piccolo, 2015). For instance, the equity measure used in facility location problems is often designed to give members of a population a fair exposure to desirable facilities such as schools, hospitals and parks, or to undesirable facilities such as nuclear power plants and oil storage tanks. Another example is to locate franchise stores in a designated region using an equity measure that assures each franchise shop will be located roughly a similar distance from the others, as a means to provide an equitable coverage of the region.

Among those problems that derive from an efficiency measure, the maximum dispersion problem and the maximum diversity problem receive the preponderance of attention in the literature (see, e.g., the survey in Martí, Gallego, Duarte, & Pardo (2013)). For the problems based on equity measures, the minimum dispersion problem (Max-Min DP) is the most widely studied equitable dispersion problem so far, addressed by a wide variety of approaches including polynomial time algorithms for special cases (Chandrasekaran & Daughety, 1981), branch and bound (Erkut, 1990), integer programming (Kuby, 1987), local search heuristics

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(Kincaid, 1992; Porumbel, Hao, & Glover, 2011), GRASP with Path Relinking (Resende, Martí, Gallego, & Duarte, 2010) and clique based heuristics (Della Croce, Grosso, & Locatelli, 2009). The Max-Mean DP has additionally been tackled via a GRASP and path relinking algorithm by Martí and Sandoya (2013).

In this paper, we focus on the Min-DiffSum DP, whose equity measure is given by $z(M) = \max_{i\in M}\sum_{j\in M} d_{ij} - \min_{i\in M}\sum_{j\in M} d_{ij}$. An equivalent binary optimization formulation arises as follows. Let $x_i = 1$ for elements $i \in M$ and $x_i = 0$ for elements $i \in N\backslash M$. Then the Min-DiffSum DP consists of assigning values 0 or 1 to each variable in $x = \{x_1, x_2, \ldots, x_n\}$, subject to m variables receiving value 1 and the other $n - m$ variables receiving value 0, to minimize the function $f(x) = \max_{i\in N:x_i=1}\sum_{j\in N} d_{ij}x_j - \min_{j\in N:x_j=1}\sum_{i\in N} d_{ij}x_i$.

Alternatively, let L_i and U_i represent lower and upper bounds on the value of $\Sigma_{j\in M}d_{ij}$, i.e. $L_i = \sum_{j\in M}\min\{d_{ij}, 0\}$ and $U_i = \sum_{j\in M}\max\{d_{ij}, 0\}$. Then a mixed linear 0–1 programming model of the Min-DiffSum DP can be formulated as follows (Prokopyev et al., 2009).

$$\min\ t = r - s \tag{1}$$

$$\text{s.t.}\quad r \geq \sum_{j\in N, j\neq i} d_{ij}x_j - U_i(1 - x_i) + M^-(1 - x_i), \quad i \in N \tag{2}$$

$$s \leq \sum_{j\in N, j\neq i} d_{ij}x_j - L_i(1 - x_i) + M^+(1 - x_i), \quad i \in N \tag{3}$$

$$\sum_{i\in N} x_i = m \tag{4}$$

$$x_i \in \{0, 1\}, \quad i \in N \tag{5}$$

where M^+ is an upper bound on the U_i values, M^- is a lower bound on the L_i values.

State-of-the-art Min-DiffSum DP algorithms

We briefly sketch the history of recent efforts to solve the Min-DiffSum DP problem as a foundation for understanding the algorithms that constitute the current state-of-the-art for this problem class. Each of the methods that follows obtains high quality results and each improves upon the algorithms that preceded it.

Prokopyev et al. (2009) proposed a greedy randomized adaptive search procedure (GRASP) which alternates between a solution construction phase and a local search phase. The solution construction phase selects an element according to a randomized greedy function to gradually enlarge the current solution until the number of elements in the solution reaches the cardinality m. The local search phase uses a descent heuristic based on swap moves. In addition, *Prokopyev* solved the mixed integer programming model of the problem by the general CPLEX optimization software. Experiments on small instances indicate that the proposed GRASP algorithm consumes less time than CPLEX and obtains high quality solutions.

Duarte, Sánchez-Oro, Resende, Glover, and Martí (2014) present several hybrid metaheuristics combining GRASP and path relinking algorithms. Two initial solution constructions are proposed, mainly differing in the way the restricted candidate list of GRASP is constructed. Three local search procedures utilizing swap moves are developed that rely on different orders for scanning the elements. The performance of the algorithm is further enhanced by using variable neighborhood search as an improvement method. Both interior path relinking and exterior path relinking schemes are investigated, in which greedy and random moves are employed to generate solutions on the path connecting the starting solution and guiding solution. Experimental analysis indicates that the hybrid variant combining the variable neighborhood search with exterior path relinking performs the best among all the proposed variants.

Mladenović, Todosijević, and Urošević (2016) present a basic variable neighborhood search (VNS) algorithm, which alternates between a local search phase and a shaking phase. With a randomized initial solution, the local search phase employs swap moves for neighborhood exploration and investigates both the first improvement and best improvement strategies. Once the local search phase is trapped in a local optimum, the shaking phase is triggered by randomly performing a sequence of swap moves to launch the search in a new region. Although the proposed VNS algorithm is quite simple, computational assessment shows that it outperforms the more complicated hybrid metaheuristics proposed in Duarte et al. (2014).

Aringhieri, Cordone, and Grosso (2014) devised a collection of innovative two-phase heuristics for the Max-MinSum DP and Min-DiffSum DP, which alternate between a constructive phase and an improvement phase. The constructive phase removes non-promising vertices or edges to quickly identify a clique of cardinality m or terminates on determining that the construction cannot find such a clique. The improvement phase modifies the subgraph inherited from the constructive phase by means of a tabu search algorithm with dynamically adjusted tabu tenure. A diversification strategy is included to drive the search to cover a large area. Experiments on six sets of benchmarks disclose the high efficacy of the proposed heuristics.

The new tabu search and memetic search algorithms

We propose two new algorithms for the Min-DiffSum DP problem, a tabu search (TS) algorithm and a memetic search (MS) algorithm, with the following features.

- The tabu search algorithm employs a neighborhood decomposition candidate list strategy to exclude the examination of unpromising moves, and to promote a more extensive neighborhood exploration in a given time period. For most instances, our candidate list strategy consumes on average one third the amount of time required by the typical approach of exploring the full neighborhood, while achieving slightly better solution quality.
- The tabu search method additionally employs a solution-based determination of tabu status which incorporates dedicated hash functions to avoid the need of tuning a tabu tenure. We show our TS solution-based strategy achieves an extensive neighborhood exploration and is more effective than the attribute-based tabu list implemented in Aringhieri et al. (2014).
- We introduce the first adaptation of memetic search for handling the Min-DiffSum DP, incorporating a distance-quality guided crossover operator to create offspring solutions with high quality and good diversity. The proposed memetic search proves particularly effective for two of the benchmark sets studied.
- Experimental assessment on seven sets of benchmarks with a total of 250 instances discloses that our tabu search algorithm finds better solutions for 179 (71.6%) of the instances and our memetic algorithm finds better solutions for 157 (62.8%) of the instances, performing statistically better than the state-of-the-art algorithms for most of the benchmark sets.

The rest of the paper is organized as follows. Sections 2 and 3 describe the proposed tabu search and memetic search algorithms, respectively. Section 4 presents experimental results and comparisons with state-of-the-art algorithms in the literature. Section 5 analyzes the influence of the essential components of our algorithms on their performance. Concluding remarks are given in Section 6.

2. Tabu search

Following the general tabu search framework (Glover & Laguna, 1997; Glover, Ye, Punnen, & Kochenberger, 2015), our tabu search