European Journal of Operational Research 000 (2016) 1–5

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Emergency relocation of items using single trips: Special cases of the Multiple Knapsack Assignment Problem

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ARTICLE INFO

Article history:
Received 29 April 2015
Accepted 1 September 2016
Available online xxx

Keywords:
Multiple Knapsack Assignment Problem
Vehicle Routing and Transportation Problem
Emergency Management
Optimization
Heuristics

ABSTRACT

The Multiple Knapsack Assignment Problem (MKAP) has been solved successfully using heuristics, but important special cases have not yet been dealt with. Motivated by an emergency relocation problem, an optimal polynomial algorithm and a high performing heuristic that take advantage of the special structure are provided for two special cases of the MKAP.

1. Introduction and related work

The Multiple Knapsack Assignment Problem (MKAP) is an NP-Hard version of the multiple knapsack problem in which the items to be assigned among the knapsacks are partitioned into disjoint sets and each knapsack may only be assigned items from one of the sets in the partition. The MKAP was introduced by Kataoka and Yamada (2014) who provide upper and lower bounds for the optimal solution and use them to develop a high-performing heuristic for solving the MKAP. However, an important special case of the MKAP remains unexplored. Specifically, this is the case where all of the items to be assigned to knapsacks are identical. To understand the relevance of this case consider the deployment of a fleet of vehicles – say, *trucks* – starting at a central location, the *depot*, to pick up items at different locations, *warehouses*, where each truck can go on at most one trip to at most one warehouse. Following pickup, the items are delivered to a new location – such as the original depot or a common transfer point for further shipment or storage.

Thinking of the trucks as knapsacks and the items at each warehouse as one of the sets in the partition of items in MKAP, the problem of assigning trips and items to trucks is equivalent to MKAP. This problem arises, for example, in a hub-and-spoke system when items are manufactured at different facilities and must be picked up periodically for delivery to a common storage facility. Another application of this problem is when items at different locations must be moved to a safe place in an emergency (such as an approaching hurricane or flood, for example). It is this second application for which the special case of identical items is particularly relevant. A common preparation for an impending disaster such as a hurricane or a flood is the evacuation of people – as was necessary in New Orleans during hurricane Katrina. Assuming each person takes up one seat in a vehicle and each person is considered to be of equal "value", then the items in this problem are identical.

Other related work regarding emergency logistic operations has focused on the reliable facility location problem (RFLP) that typically takes the form of a two-stage problem. In the first stage facility locations are selected and in the second stage it is randomly determined which facilities are operational and customers are assigned to the operational facilities. RFLP was initially studied with uniform failure probabilities at each location (Snyder & Daskin, 2005; 2006) and then generalized to allow for site-dependent failure probabilities (Aboolian, Cui, & Shen, 2013; Berman, Krass, & Menezes, 2007; Cui, Ouyang, & Shen, 2010; Lei & Tong, 2013; Li, Zeng, & Savachkin, 2013; Lim, Daskin, Bassamboo, & Chopra, 2010; Shen, Zhan, & Zhang, 2011). These initial models all assumed uncapacitated facilities, where there was no limit on the demand any single facility could handle. There have also been a few studies dealing with the capacitated RFLP (Aydin & Murat, 2013; Peng, Snyder, Lim, & Liu, 2011). However, to date no consideration has

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http://dx.doi.org/10.1016/j.ejor.2016.09.004
0377-2217/

yet been given to reallocation of capacity – in the form of people, inventory and/or equipment – across facilities before a disruptive event occurs. For instance, this would be the case if the disruption were a weather-related event. In such a case, people, inventory and/or equipment could be repositioned from high-risk to low-risk locations in anticipation of the event through the use of the models considered here.

To frame this as a general problem, consider a fleet of trucks located at various depots that need to pick up a number of identical items at several warehouses. A standard transportation problem (TP) (Hillier and Lieberman, 2010) would determine how to use the trucks to pick up the items at the warehouses so as to minimize the total travel distance. However, what distinguishes this problem from a standard TP is that the objective is to maximize the number of items the fleet can pick up when there are not enough trucks to pick up all the items. Because travel distance is not the objective, it can be assumed that all trucks are located at a single depot and the goal is to determine how to deploy those trucks to pick up the maximum number of items from the warehouses.

2. Problem descriptions and solution methodologies

In this section, two special cases of MKAP dealing with identical items are formulated and solved. It is important to note that the formulations presented here are special cases of MKAP in which the special structure is exploited to develop alternative algorithms. Specifically, a polynomial algorithm is provided for solving the first special case. The second special case, however, is NP-complete and so heuristics based on the special structure are developed and computational results are presented to show the efficiency and quality of the solutions obtained when compared to solving the same problem as a general MKAP.

2.1. The single-type-of-truck/single-type-of-item problem

The first, and most simple, version of the problem is stated as follows.

The STSI problem: Given a fleet of T trucks located at a depot, each capable of holding C items of the same type, and given D_j of these items at warehouses $j = 1, \dots, N$, the goal is to determine how to deploy the trucks on a single trip to pick up the most number of items possible at the warehouses, with each truck going to at most one warehouse.

This version of the deployment problem can be solved using sorting. However, for completeness and to lay the foundation for more complex versions of the problem, the solution is discussed in some detail. To solve the STSI problem, it is necessary to decide the number of trucks, t_j, to send from the depot to warehouse j. When those t_j trucks arrive at warehouse j they can collectively hold up to Ct_j of the D_j items at that warehouse. As a result, the STSI problem is to find values for the variables:

t_j = the number of trucks to send from the depot to warehouse j $(j = 1, \dots, N)$

so as to

$$\begin{aligned} \max \quad & \sum_{j=1}^{N} \min\{Ct_j, D_j\} \\ \text{s.t.} \quad & \sum_{j=1}^{N} t_j \le T \\ & \text{all } t_j \ge 0 \quad \text{and integer} \end{aligned} \tag{1}$$

It is possible to solve the model in (1) using sorting in $O(N\log(N))$ time as follows. First, compute the following two numbers for each warehouse $j = 1, \dots, N$:

$\underline{T}_j = \left\lfloor \dfrac{D_j}{C} \right\rfloor$ = the maximum number of full truckloads of items available at warehouse j

$\overline{T}_j = \left\lceil \dfrac{D_j}{C} \right\rceil$ = the minimum number of trucks needed to pick up all items at warehouse j.

There are two easy cases: (1) if $T \ge \sum_j \overline{T}_j$, then there are enough trucks to pick up all the items at all warehouses, and (2) if $T \le \Sigma_j \underline{T}_j$, then each truck can be filled to capacity. The interesting situation is when $\sum_j \underline{T}_j < T < \sum_j \overline{T}_j$. In this case, the optimal solution is to fill the first $\Sigma_j \underline{T}_j$ trucks to capacity and then use the remaining $\hat{T} = T - \sum_j \underline{T}_j$ trucks to pick up the leftover items at the $\hat{T}$ warehouses having the most number of remaining items. To that end, sort the warehouses so that

$$D_1 \bmod C \ge D_2 \bmod C \ge \cdots \ge D_N \bmod C, \tag{2}$$

and send the remaining trucks to the first $\hat{T}$ warehouses that have leftover items to be picked up. In the resulting solution, the first $\Sigma_j \underline{T}_j$ trucks are filled to capacity and the remaining $\hat{T}$ trucks are carrying the maximum number of leftover items possible. The running time of $O(N\log(N))$ is based on the time needed to sort the N warehouses according to (2).

The following example illustrates this algorithm. Suppose the depot has $T = 12$ trucks that can each hold $C = 50$ items together with the following data for the warehouses (which are sorted according to (2) in decreasing order of the number of leftover items):

Warehouse	**1**	**2**	**3**	**4**	**5**	**6**	**7**
D_j	38	134	78	126	22	157	50
$\overline{T}_j$	1	3	2	3	1	4	1
$\underline{T}_j$	0	2	1	2	0	3	1
$D_j \bmod C$	38	34	28	26	22	7	0

In this example $\sum_j \underline{T}_j = 9 < T = 12 < 15 = \sum_j \overline{T}_j$ and so the optimal solution is to fill the first 9 trucks to capacity at warehouses 2, 3, 4, 6, and 7 and then send the remaining three trucks to pick up the 38, 34, and 28 items left over in warehouses 1, 2, and 3 respectively.

2.2. The multiple-type-of-truck/single-type-of-item problem

Building on top of STSI, in the second special case of MKAP, the fleet consists of trucks with different capacities but the items are still identical.

The MTSI problem: Given a fleet of T_i trucks of type i, for $i = 1, \dots, M$, in which each truck of type i is capable of holding C_i items, and given D_j items at warehouses $j = 1, \dots, N$, the goal is to determine how to deploy the trucks on a single trip to pick up the most number of items possible at the warehouses, with each truck going to at most one warehouse.

To solve the MTSI problem, it is necessary to decide the number, t_{ij}, of trucks of type i to send from the depot to warehouse j. When those t_{ij} trucks arrive at warehouse j they can collectively hold up to $\Sigma_i C_i t_{ij}$ of the D_j items at that warehouse. As a result, the MTSI problem is to find the values for the variables:

t_{ij} = the number of trucks of type i to send from the depot to warehouse j $(i = 1, \dots, M;\ \ j = 1, \dots, N)$