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Efficiency decomposition and aggregation in network data envelopment analysis

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ABSTRACT

Network data envelopment analysis (DEA) is extended from conventional DEA to explore the internal structure of network production systems so that the efficiencies are measured more appropriately. There are two approaches for discussing the relationships between the system and division efficiencies, efficiency decomposition and efficiency aggregation. This paper develops a relational model from the viewpoint of efficiency decomposition to derive the relationships between the system and division efficiencies for network systems. The system efficiency can be decomposed into a weighted average of the division efficiencies adjusted by a factor, and the efficiency of the system is less than the weighted average of those of the divisions. The Major League Baseball example that has been discussed in the literature is used to illustrate the idea presented in this paper. The results of the efficiency decomposition model are compared to those obtained from an efficiency aggregation model that defines the aggregate efficiency of the divisions to be the system efficiency. This system efficiency is found to be very close to a measure contained in the proposed model. The proposed model can thus be used for both efficiency decomposition and aggregation.

1. Introduction

Efficiency measurement is an important task in management, as it not only shows the past accomplishments of a unit, but also indicates directions for future development. Since the seminal work of Charnes, Cooper, and Rhodes (1978), data envelopment analysis (DEA) has become an effective technique for measuring the relative efficiency of a set of decision making units (DMUs) that apply multiple inputs to produce multiple outputs. In its original settings only the inputs consumed by and the outputs produced from the system are considered, and the operations of the internal components are ignored in measuring efficiencies. When a system is composed of several components operating interdependently, it has been found that ignoring the operations of the components may produce efficiency measures that are misleading (Castelli, Pesenti, & Ukovich, 2004, Kao & Hwang, 2010). The operations of the components should thus be considered whenever the data is available. The DEA methodology applied to systems for which the internal structure is considered is named network DEA by Färe and Grosskopf (2000), and many related models and applications have been developed (Kao, 2014b).

A typical example of efficiency measurement in network systems is the investigation of the contribution of IT (information technology) to the performance of banks. The conventional whole-unit DEA models may find that IT does not affect the performance of banks. However, if we separate the whole operation of banking and similar industries into two stages, capital collection and investment, it will then be clear that while IT is useful for the former, whether the firms would actually make a profit or not is dependent on correct investment decisions being made. This indicates that to study the performance of a DMU, it is necessary to study its component operations, so that the cause of any inefficiency can be identified.

The feature that characterizes the network system is the intermediate product. Different from the exogenous inputs that are supplied from outside and the final outputs that are produced for outside, the intermediate products are produced and consumed within the system, and are thus not visible from outside. By considering the component divisions as independent DMUs, their efficiencies can be calculated from the inputs they consume and the outputs they produce. The major objective of efficiency measurement for a network system is to identify the most influential divisions that have decisive effects on the overall efficiency of the system, such that improving the efficiency of these divisions will improve the efficiency of the system the most. To achieve this it is desirable to find the relationships between the system and division efficiencies.

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There are two approaches for this, efficiency decomposition and efficiency aggregation. The efficiency decomposition approach measures the system efficiency from the inputs it consumes and the outputs it produces, and then derives the relationships. The efficiency aggregation approach, in contrast, defines the relationships first, and then measures the system and division efficiencies based on these. For both approaches the system efficiency is a function of the division efficiencies. The difference is that the system efficiency in the decomposition approach does not involve the intermediate products produced and consumed in the system, and its relationships with the division efficiencies are to be explored. The system efficiency in the aggregation approach is artificially aggregated from the division efficiencies following a pre-specified mathematical relationship. While efficiency aggregation is always possible, as long as a mathematical form is given, efficiency decomposition is not so straightforward, and only limited cases have been reported. In this paper a systematic way for decomposing the system efficiency into division efficiencies is introduced. It is applicable to most network systems, except those with feedbacks and cycles.

This paper is organized as follows. Sections 2 and 3 introduce the ideas of efficiency decomposition and efficiency aggregation, respectively, in network systems. How to decompose the system efficiency into division efficiencies for network systems in a systematic way is discussed in Section 4. Section 5 provides an example to illustrate the differences between the efficiencies measured from these two approaches. Finally, in Section 6, some conclusions are drawn, and some directions for future research are suggested, based on the discussions in the preceding sections.

2. Efficiency decomposition

Consider a system composed of q components. In the literature the terms sub-unit, sub-DMU, sub-process, and division have been used for such components, and in this paper we use the term division when there is no ambiguity. Denote $X_{ij}^{(k)}$ and $Y_{rj}^{(k)}$ as the ith input , $i\in I^{(k)}$, supplied from outside to and the rth final output , $r\in O^{(k)}$, produced from the kth division, k=1,…, q, of the jth DMU, j=1,…, n, respectively, where $I^{(k)} \subset\{1, 2,\ldots, m\}$ and $O^{(k)} \subset\{1, 2,\ldots, s\}$ are the index sets for the inputs and outputs of division k, and there are m inputs and s outputs in all q divisions. Further, denote $Z_{dj}^{(a,b)}$ as the dth intermediate product produced by division a for division b to use. $\sum_{a=1}^{q} Z_{fj}^{(a,k)}$ is then the total amount of the fth intermediate product, $f\in M^{(k)}$, produced by other divisions for division k to use, and $\sum_{b=1}^{q} Z_{gj}^{(k,b)}$ is the total amount of the gth intermediate product, $g\in N^{(k)}$, produced by division k for other divisions to use, where $M^{(k)} \subset \{1, 2,\ldots, h\}$ and $N^{(k)} \subset\{1, 2,\ldots, h\}$ are the index sets for the intermediate products to be consumed and produced by division k, respectively, and there are h intermediate products in all q divisions. Fig. 1 shows a general structure for network systems.

Theoretically every division can consume all inputs and intermediate products, and produce all outputs and intermediate products. In reality, however, a division will consume only certain inputs and intermediate products, and produce certain outputs and intermediate products. To make the expression more systematic, we set $X_{ij}^{(k)}=0$ for $i\notin I^{(k)}$, $Y_{rj}^{(k)}=0$ for $r\notin O^{(k)}$, $Z_{fj}^{(a,k)}=0$ for $f\notin M^{(k)}$, and $Z_{gj}^{(k,b)}=0$ for $g\notin N^{(k)}$. Following the convention of denoting X_{ij} as the ith input consumed and Y_{rj} as the rth output produced by the jth DMU, we have $X_{ij}=\sum_{k=1}^{q} X_{ij}^{(k)}$ and $Y_{rj}=\sum_{k=1}^{q} Y_{rj}^{(k)}$. The conventional DEA model, which considers only the inputs and outputs of the system, under constant returns to scale is:

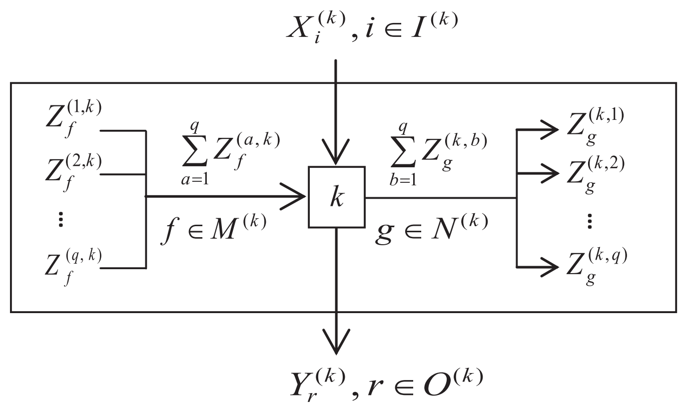

Fig. 1. General network systems.

$$\begin{aligned} \max.\ & \sum_{r=1}^{s} u_r Y_{r0} \Big/ \sum_{i=1}^{m} v_i X_{i0} \\ \text{s.t.}\ & \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0,\ j=1,\ldots,n \\ & u_r,\ v_i \ge \varepsilon,\ r=1,\ldots,s,\ i=1,\ldots,m \end{aligned} \tag{1}$$

where ε is a small non-Archimedean number (Charnes & Cooper, 1984) imposed upon all multipliers to avoid ignoring unfavorable factors in measuring efficiencies. In this model the operations of the component divisions are ignored.

The main concerns of the top management in efficiency measurement are the inputs consumed by the system and the outputs it produces. More specifically, it is desired to use the least amount of inputs to produce the current amount of outputs, or to use the current amount of inputs to produce the largest amount of outputs. The divisions in the system must cooperate with each other to achieve this goal, and the relational modeling approach (Kao, 2009a) which requires the same factor to have the same multiplier, regardless of the division it is associated with, or the role that the associated factor plays, i.e. an input or an output, is used for this. In addition to the aggregate operations of the system, the operations of individual divisions must also be taken into account. The efficiency decomposition model for measuring the system efficiency under constant returns to scale can be formulated as:

$$\begin{aligned} E_0 = \max.\ & \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{r0}^{(k)} \Big/ \sum_{k=1}^{q}\sum_{i=1}^{m} v_i X_{i0}^{(k)} \\ \text{s.t.}\ & \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_i X_{ij}^{(k)} \le 0,\ j=1,\ldots,n \\ & \left[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g\left(\sum_{b=1}^{q} Z_{gj}^{(k,b)}\right)\right] \\ & \quad - \left[\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f\left(\sum_{a=1}^{q} Z_{fj}^{(a,k)}\right)\right] \le 0, \\ & k=1,\ldots,q,\ j=1,\ldots,n \\ & u_r, v_i, w_g \ge \varepsilon,\quad \forall r,i,g \end{aligned} \tag{2}$$

The objective function of Model (2) is the system efficiency, expressed as the ratio of the aggregate output produced to outside to the aggregate input supplied from outside. Each constraint in the first and second constraint sets corresponds to one DMU and one division, respectively. Note that the same factor, input X, output Y, or intermediate product Z, has the same multiplier v, u, or w associated with it, no matter which division they correspond to, and