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Decision Support

Super efficiency evaluation using a common platform on a cooperative game

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ABSTRACT

Classical DEA models (such as CCR and BCC models) compute efficiencies of decision-making units (DMUs) based on a common efficient frontier but with no capability of differentiating efficient units. The super efficiency model was developed to rank efficient DMUs based a new efficient frontier comprised by all other DMUs. This may lead to a multi-platform problem that different efficient DMUs are evaluated based on different efficient frontiers and the resulted super efficiencies of efficient DMUs are not comparable. This paper addresses the multi-platform problem from the perspective of a cooperative game. Efficient DMUs are regarded as players and subsets of these efficient DMUs as coalitions. The effect of a coalition on a specific efficient DMU is defined as the DMU's efficiency change proportion (ECP) based on the traditional DEA models when the coalition is removed from the reference set. Basing on the ECP, we define a characteristic function as the sum of all efficient DMUs' ECPs, and prove that this function is super-additive. Then, the Shapley value is introduced as a solution of this cooperative game and applied to rank efficient DMUs. The proposed approach is demonstrated by two numerical examples. Finally, we extend the proposed approach in this paper to the VRS assumption.

1. Introduction

Originating from the work of Farrell (1957) and Debreu (1951), data envelopment analysis (DEA) developed by Charnes, Cooper, & Rhodes (1978) is a popular nonparametric method to evaluate performance of various peer decision-making units (DMUs). Now, DEA has been widely applied to measuring performance in many areas such as universities (Lim & Zhu, 2013), banks (Staub, e Souza, & Tabak, 2010; Wanke, Barros, & Emrouznejad, 2016), reallocation of emission permits (Wu, Du, Liang, & Zhou, 2013), and health care (Pulina, Detotto, & Paba, 2010; Shwartza, Burgess, & Zhu, 2016). Its popularity can be attributed to no requirements specific production functional form and prior knowledge of DMUs' input and output weights (Cooper, Seiford, & Tone, 2007; Cooper, Seiford, & Zhu, 2011). In our opinion, another reason of its popularity is that DMUs are evaluated fairly under a common frontier comprised of all DMUs, and such a frontier constitutes a common platform for all DMUs. Therefore, efficiencies obtained by classical DEA models are comparable and can be used to rank DMUs. However, classical DEA models fail to rank (weakly) efficient DMUs[1], for all efficient DMUs' efficiencies are ones.

A number of novel approaches are proposed to rank efficient DMUs (Angulo & Lins, 2002; Adler, Friedman, & Stern, 2002). For example, super efficiency, introduced by Peterson and Anderson (1993), is one of the important approaches for ranking efficient DMUs. In this approach, each efficient DMU is evaluated by an efficient frontier comprised by all other DMUs. If a DMU is strongly efficient, it may be beyond of envelopment of such the frontier. Therefore, the efficient DMU's super efficiency may be bigger than one. In this case, super efficiency is a popular approach used to rank efficient DMUs. And it has been widely applied in many areas such as fixed cost allocation (Li, Yang, Liang, & Hua, 2009), manufacturing firm evaluation (Düzakın & Düzakın, 2007), and bank evaluation (Avkiran, 2011). However, this model brings at least two problems. The first is that the model may be infeasible based on the variable return to scale (VRS) assumption

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[1] In this paper, "efficient DMUs" refer to DMUs that are on the efficient frontier. Therefore, for "radial" DEA models, the set of "efficient DMUs" not only includes strongly Pareto-efficient DMUs but also weakly efficient DMUs. For non-radial DEA models, such as the slack-based measure model, "efficient DMUs" only refer to Pareto-efficient DMUs.

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(Baker & Talluri, 1997; Seiford & Zhu, 1999; Cook, Liang, Zha, & Zhu, 2008; Johnson & McGinnis, 2009; Lee, Chu, & Zhu, 2011; Chen & Liang, 2011; Lee & Zhu, 2012; Chen, 2013). The second is the multi-platform problem. Super efficiency model evaluates each efficient DMU by a specified efficient frontier comprised of other DMUs, and such the frontier may vary when different efficient DMUs are evaluated. Therefore, super efficiency evaluates these efficient DMUs based on different standards, and super efficiencies of efficient DMUs may be not comparable. For details, readers can refer to Section 3. Similarly, Banker & Chang (2006) argued that the super efficiency model is useful for outlier identification but not good at ranking efficient units. Banker, Chang, & Zheng (2015) found that the reason of unsatisfactorily comes from the "left corner" of DMUs.

Based on the philosophy of super efficiency, a novel approach of changing the reference set is proposed to discriminate efficient DMUs (Chen & Deng, 2011; Jahanshahloo, Junior, Lotfi, & Akbarian, 2007; Hibiki & Sueyoshi, 1999). This approach regards each efficient DMU as a candidate. For a given candidate, its effect is defined as the sum of the efficiency changes of other DMUs when the candidate is omitted from the reference set. Such an effect is called the DEA cross reference (DCR). This approach ranks each efficient DMU based on its DCR value. The higher its DCR value, the higher its rank. However, the nature of this approach is the same as the super efficiency, and it still has the multi-platform problem.

Cross-efficiency is another important approach of ranking efficient DMUs. It is first proposed by Doyle & Green (1994) and developed by Liang, Wu, Cook, & Zhu (2008a), Wu, Liang, & Yang (2009), Wu, Liang, & Chen (2009), Wu, Liang, Yang, & Yan, (2009), Alcaraz, Ramón, & Ruiz (2013), Ruiz (2013) and Wu, Chu, Sun, & Zhu, (2016). This approach allows each DMU to selfishly choose an optimal set of input and output weights to the classical DEA, and defines the average value of a DMU efficiencies based on these optimal weights as its cross efficiency. However, cross-efficiency scores are generally not unique, and depend on which of the alternate optimal solutions to the DEA linear programs is used (Liang, Wu, Cook, & Zhu, 2008b).

Common weight DEA is also an approach for ranking DMUs (Cook, Roll, & Kazakov, 1990; Roll, Cook, & Golany, 1991; Kao & Hung, 2005; Wu, Liang, & Yang, 2009; Kao, 2010; Wu, Chu, Zhu, Li, & Liang, 2016). This approach attempts to find a common set of weights to calculate efficiencies for all DMUs, and ranks them based on their efficiencies. Compared to the cross efficiency, this approach evaluates DMUs based on a common platform (a common set of weight), and the obtained efficiencies are comparable. However, it's difficult to find a comprise principle to choose the common set of weights and different principles adopted lead to different common sets of weights chosen as well as ranks among DMUs.

Besides, Salo & Punkka (2011) introduced the efficiency dominance to rank DMUs by considering all possible input and output weights: a DMU is only said to be better than another DMU if its efficiency score is higher for all feasible input and output weights. This approach is fair to rank DMUs but lacks of discrimination for efficient DMUs.

In the current study, we approach the multi-platform problem in efficiency evaluation from the perspective of cooperative game theory. Efficient DMUs are assumed to compete and cooperate with each other in the evaluation. For example, in bank performance evaluation, efficient branches compete with other branches to be the best benchmark because branches and their managers with higher ranks often receive more rewards and support in the future. On the contrary, efficient branches cooperate with each other to construct a common efficient frontier to evaluate inefficient ones, and make the evaluate result comparable.

Combining DEA with game theory is not rare in DEA literature, Nakabayashi & Tone (2006) presented a cooperative game to solve the problem of consensus-making between unit and organization when evaluating performance via DEA models. Li & Liang (2010) used a cooperative game to select indices for DEA. Some researches focused on incorporating game theory with a two-stage DEA model (Zhu, 2004; Liang, Cook, & Zhu, 2008; Zha & Liang, 2010; Du Liang Chen Cook & Zhu, 2011; Li, Chen, Liang, & Xie, 2012; Zhou, Sun, Yang, Liu, & Ma, 2013). Lozano (2012; 2013), Lozano, Mármol, & Hinojosa (2015), Borrero, Hinojosa, & Mármol (2016) and Wu, Zhu, Cook, & Zhu (2016) proposed approaches based on cooperative games in dealing with problems of information and technology sharing among organizations.

The purpose of this paper is to rank efficient DMUs with a common platform. We define a cooperative game with transferable utility (TU-game) to construct the common platform for efficient DMUs. Each efficient DMU is regarded as a player, each possible subset of efficient DMUs is regarded as a coalition. The effect of a coalition on each efficient DMU is defined as its efficiency change proportion (ECP) when the coalition is removed from the reference set. Basing on ECP, we define a characteristic function of the coalition as the sum of the ECPs of all efficient DMUs. We also prove that the characteristic function satisfies super additivity. Then, with the fairness axioms of efficiency and dummy player, symmetry and additivity, a Shapley value is used as a solution to the cooperative game and to rank DMUs. In this paper, we will also extend the proposed approach under the variable returns to scale (VRS) assumption.

The remainder of this paper is organized as follows. Section 2 describes the problem; Section 3 defines ECP to measure the marginal role of a DMU set on an efficient unit; Section 4 presents a common platform approach based TU-game for ranking efficient DMUs; Numerical examples are presented in Section 5; Section 6 extends our common platform approach to the VRS assumption; The conclusion is elaborated in Section 7.

2. Problem description

In this paper, we use the following notations:

Table 1
Notation explanation.

N	Original DMU set
E^*	The set of DMUs which are on the efficient frontier
S	A subset of E^*, corresponding to a coalition in this paper
j, f	Any DMU in original DMU set $N = \{1,2,\ldots,n\}$
k, d	An efficient DMU in set E^*
I	Index on inputs, $i = 1,2,\ldots,m$
R	Index on outputs, $r = 1,2,\ldots,h$
x_{ij}	Amount of input i consumed by DMU j
y_{rj}	Amount of output r produced by DMU j

Notably, E^* defined in Table 1 is the set of DMUs that are on the efficient frontier. Therefore, their efficiencies are all 1. For the "radial" DEA models in this section, E^* may not only include strongly efficient DMUs, but also include weakly efficient DMUs. Suppose there are n independent DMUs in set N, and each DMU_j ($j \in N = \{1, 2, \ldots, n\}$) consumes m inputsx_{ij}($i \in M = \{1, 2, \ldots, m\}$) to generate h outputsy_{rj}($r \in H = \{1, 2, \ldots, h\}$). The efficiency rating for any given DMU_f can be computed by using the standard input-oriented CCR model (Charnes, Cooper, & Rhodes, 1978) as follows:

$$\begin{aligned} & E_f(N) = Min \quad \theta \\ s.t. \ & \sum_{j\in N} \lambda_j x_{ij} \le \theta x_{if}, i \in M \\ & \sum_{j\in N} \lambda_j y_{rj} \ge y_{rf}, r \in H \\ & \lambda_j \ge 0, \forall f, j \in N, \theta : free \end{aligned} , \tag{1}$$

where $E_f(N)$ is the efficiency for DMU_f based on reference set N.