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Decision Support

More possessions, more worry☆

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ABSTRACT

A common wisdom asserts that the wider the universe of assets to choose from, the greater the investor's welfare. We show that this is not the case in practice, where parameters have to be estimated even when the estimates are unbiased. Surprisingly, risk aversion plays a crucial role corresponding to the desirability of asset expansion by dividing investors in three groups: investors with very low risk tolerance and investors with very high risk tolerance are better-off with asset expansion, and investors with moderate risk tolerance are worse-off despite the option to refrain from investing in the additional asset.

1. Introduction

It is well known that investors commonly hold an average of two to four different stocks in their portfolios (Barber & Odean, 2001; Blume, Crockett, & Friend, 1974). These findings have direct theoretical and empirical implications to asset pricing with deviations from the CAPM. Specifically, due to the underdiversified nature of these equity portfolios, there is increasing evidence that nonmarket risk may play an important role in determining the cross-section expected stock returns (see, e.g., Bali, Cakici, & Whitelaw, 2011, 2014). This observed deviation from the CAPM, by which all available assets should be held in the portfolio, led to the development of segmented CAPM, also called General CAPM (GCAPM) (Levy, 1978; Merton, 1987).

We suggest in this paper another research path that rationalizes the optimal holding of a relatively small number of assets in the portfolio on the one hand, and on the other hand, justifies the existence of various funds with management fees even if those funds demonstrate no professional ability to outperform the market. We show that the welfare of some investors, albeit not all of them, may improve by holding a relatively small number of assets in their portfolios in the practical scenario where the parameters are unknown and have to be estimated.[1] Thus, holding a small number of assets for some investors is optimal even with no transaction costs and other economic factors needed to justify GCAPM.

Our model does not contradict the previous suggested models but rather suggests another possible avenue in explaining the relatively small number of assets held in the typical portfolio. Our model also offers one additional implications and economic insights. Specifically, unlike the previous studies our results shed light on the relation between the degree of risk aversion and the motivation to hold small portfolios, as the desirability for asset expansion depends on the risk tolerance of the investor under consideration. Thus, while in the Levy-Merton segmented equilibrium model (the GCAPM) the number of assets held in the portfolio does not depend on the investor's risk tolerance, in the suggested model given in this paper risk aversion plays an important role. Some investors are better-off with small portfolios and others will optimally include many assets in their portfolios—a result that fits the ample empirical observations well.

Our claim is somewhat counterintuitive. Suppose that one has to choose an investment portfolio from a set of k available assets (denoted by S_k) or alternatively from a set of K available assets (denoted by S_K), where $K > k$ and $S_K \supseteq S_k$. Intuitively, the investor cannot be worse-off by the expansion of the universe of available

☆ **Ethics of Our Fathers 2:7.**

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[1] We analyze in this paper the effect of estimation errors on the number of assets included in the portfolio. The effect of possible estimation errors on investment weights and on equilibrium prices is well known (see for example, Barberis, 2000; Best & Grauer, 1991; Chopra & Ziemba, 1993; Levy & Roll, 2010). Several methods are suggested to protect the investor from sampling errors; most of these methods either adjust the sample parameters or impose some constraints on the investment weights (see for example, Black & Litterman, 1992; DeMiguel, Garlappi, Nogales, & Uppal, 2009; Frost & Savarino, 1988; Green & Hollifield, 1992; Jagannathan & Ma, 2003; Jorion, 1986; Ledoit & Wolf , 2003; Markowitz & Usmen, 2003; Michaud, 1989;). We employ the simple unbiased estimate of the parameters as there is no agreement in the literature on the best method to protect against sampling errors (see Levy & Levy, 2014). Our method can be expanded to incorporate constraints on the weight of each asset in the portfolio.

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assets as she always has the option to refrain from investing in the added assets. This assertion is absolutely true and conforms to one's intuition when the various parameters are known. However, we show in this study that in the most important and practical case where the various parameters of the assets under consideration are unknown the investor may be worse-off when the universe of assets expands. The reason for the possibility of being worse-off is that even if the estimates are unbiased, the possible estimation errors may distort the optimal diversification; hence, welfare may decline with an asset expansion. In a nutshell, one has to weigh the advantage of asset expansion versus the disadvantage of introducing more possible estimation errors.

In the mathematical analysis, we compare a situation with $k = 1$ to the situation where $K = 2$ with estimation risk. We can compare also the case of k assets to the case of $k + 1$ assets for any number $k > 1$. Yet, it is well known that generally the marginal benefit from diversification decreases with an increase in n; hence, if with the expansion from one to two assets welfare decreases due to the unknown parameters, it presumably decreases *a fortiori* by the expansion from n to $n + 1$ assets where $n > 1$. The advantage of our setting is the transparent presentation with an intuitive explanation.

Specifically, we assume holding one risky asset (which can be an ETF on the market portfolio or on some index, such as the S&P 500 index) and analyze the effect of adding a second, riskless asset.[2] We show that with possible estimation errors, some investors may be worse-off and some better-off by adding one more asset to the portfolio. We divide the investors into groups based on the degree of their risk aversion. Generally, economics and finance models divide investors into two groups classified by low and high risk-aversion parameters. Therefore, the second surprising result emerging from this study is that we obtain three groups of investors: Denoting with A_i, the risk-aversion parameter, we have the following results: for the range of risk aversion $A_L < A_i < A_U$ the investors are worse-off by the addition of the riskless asset, and for any risk-aversion parameter falling out of this range the investor is better-off, where A_L and A_U are the lower bound and the upper bound of the risk-aversion parameter; they are functions of the Sharpe index, which we will derive in this paper.

It is well known that in practice some investors neither borrow nor lend money at the riskless rate, which conforms to our theoretical findings but contradicts the CAPM. Presumably, the main reason for this phenomenon is that some investors simply have no access to the risk-free asset or that the borrowing rate is higher than the lending rate, which justifies theoretically that for some segment of investors neither borrowing nor lending is optimal. We add here one more possible explanation, asserting that for a segment of investors, borrowing or lending at the riskless interest rate decreases welfare; hence, these investors do not wish to add the riskless asset even when it is available to them and the borrowing rate is equal to the lending rate. We also predict in this study which type of investors fall in this category (see Eq. (22)), but conducting an empirical study verifying our reason for denying the riskless asset is very complex as one needs not only to examine the composition of each individual's portfolio, one needs also to learn the risk tolerance of the investor under consideration—quite a difficult task.

Obviously, the estimation errors depend on the selected estimation method as well as on the number of available observations. As the estimates employed in this study are unbiased, statistical theory advocates that as the sample size $n \to \infty$, the estimation errors approach zero and our paper is redundant, as we are back in classical portfolio theory with known parameters. Moreover, nowadays there is ample available data supporting the above claim. However, suppose that one wishes to estimate the mean return of IBM. The IBM of the 1960s is much different than the IBM of today, and as we would like to estimate the future mean return, it is obvious that relevant data for our purpose is limited. Some may argue that the availability of ample, high-frequency, intraday data provides us with a lot of observations that can remedy the estimation errors. However, intraday price changes reflect short-term-liquidity imbalances and carry little information, in our example, on the long-run distribution of IBM returns. Moreover, as the returns are non-additive, the assumed investment horizon dictates the number of observations because if the planned investment horizon is, say, 1 year, one needs to use annual data to avoid biases in the estimates as well as biases in equilibrium pricing (for more detail on the potential biases induced by selecting the wrong investment horizon, see Levhari & Levy, 1977). Specifically, one cannot increase the number of observations by shifting, say, from annual data to monthly data when the relevant investment horizon is 1 year. Hence, the number of useful observations is limited and therefore the estimation errors exist, which is a basic ingredient of our paper.

The structure of this paper is as follows. In Section 2 we provide the model setting, assumptions, and a graphical analysis conveying the main idea of this paper. In Section 3 we analyze the sampling estimation errors effect in the expected utility framework with risk aversion. Sections 4 and 5 provide the asset expansion effect in the mean-variance (MV) framework with full information and with sampling errors, respectively. Section 6 concludes.

2. The model setting and graphical demonstration of the main idea

2.1. The model setting

Suppose that the investor with a given known utility function invests in one asset and considers diversifying in a second asset where the true distributions are unknown and have to be estimated. For example, in the mean-variance (MV) framework, one or more of the parameters may be unknown; hence, one needs to estimate these parameters. For simplicity of the discussion, assume first that the variances and correlations are known but the means are unknown (in the graphical illustration, we also analyze the case where the variance is unknown). Emphasizing this scenario is motivated by the fact that it is much easier to obtain accurate estimates of the variances and covariances than the expected values (see Kogan & Wang, 2002; Merton, 1992). Thus, we assume the investor does not know the true means and hence makes investment decisions based on the sample means. Relying on the sample means, the investor may decide to diversify between the two assets and for the selected portfolio she can calculate her estimated (or subjective) expected utility, which obviously depends on the adopted estimation method by the individual under consideration.

Holding initially one asset and deciding to add a second asset to the portfolio, the investor thinks she increases expected utility, otherwise she would not diversify. However, this is not the *true* expected utility as the true expected utility, for the given selected investment weights, can be calculated only with the true unknown means. The investor may select investment weights that are substantially different than the optimal investment weights; hence, the true expected utility may be smaller than the expected utility with holding one asset only, although the investor thinks the expected utility increases by adding the second asset to the portfolio. As the probability of no estimation error in the continuous

[2] If we start with the riskless asset and analyze whether the utility increases or decreases by adding the risky asset, we find that all investors may be worse-off or better-off by asset expansion, depending on the Sharpe ratio and the sample size. Thus, even in this setting asset expansion does not necessarily increase utility.