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A note on “A multi-period profit maximizing model for retail supply chain management”

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ABSTRACT

In this note we present an efficient exact algorithm to solve the joint pricing and inventory problem for which Bhattacharjee and Ramesh (2000) proposed two heuristics. The algorithm is based on a method proposed by Thomas (1970) and we show additional properties which can be used to arrive at an even more efficient algorithm. Furthermore, we point out several shortcomings in the paper by Bhattacharjee and Ramesh.

1. Introduction

Bhattacharjee and Ramesh (2000) consider a joint pricing and inventory model for a monopolistic retailer who is dealing in a single product. In the pricing and inventory problem, the retailer wants to maximize his profit considering revenue and all relevant costs for a given planning horizon. Bhattacharjee and Ramesh propose two heuristic algorithms and an exact approach that runs in exponential time to solve this problem. According to Google Scholar the paper has received more than 100 citations, including recent ones, implying that the topic is still of interest. Instead of solving the problem either inefficiently or heuristically, we show in this note that the problem can be solved to optimality in polynomial time, that is, in an efficient way. We do this by applying a method already proposed by Thomas (1970) for a similar problem. Furthermore, we prove additional properties of optimal solutions, which can be used to arrive at an even more efficient algorithm. Finally, we point out shortcomings in the modeling and analysis in the paper of Bhattacharjee and Ramesh.

Although there are similar models in the literature, to the best of our knowledge, the lot-sizing and pricing model under consideration is not a special case of any other existing model, implying that our results cannot be directly obtained from the existing literature. To position our work, we briefly describe some related works from the literature. Deng and Yano (2006) consider a joint lot-sizing and pricing model with production capacities. Although the model of Deng and Yano (2006) is more general in terms of capacities, it does not consider lower and upper bounds on the prices. Furthermore, Geunes, Romeijn, and Taaffe (2006) consider a model with a set of customers in each period. Each customer demand can be partly served and the revenue of a customer depends linearly on the amount served, which leads to a piecewise linear concave revenue function. Again, this is different from the model under consideration, where the revenue function does not have this particular structure. We note that the approach of Geunes et al. (2006) could be used to solve the problem under consideration by approximating the revenue function by a piecewise linear function, but this would result in a loss of precision and efficiency.

The remainder of this note is organized as follows. In Section 2 we describe the joint pricing and inventory model and we give a mathematical formulation. In Section 3 we present the exact method proposed by Thomas (1970), apply this method to the Bhattacharjee and Ramesh case, and show how the running time can be further improved. In Section 4 we point out the shortcomings in the main results presented by Bhattacharjee and Ramesh.

2. Problem description

Bhattacharjee and Ramesh consider the following joint pricing and inventory model. There is a monopolistic retailer dealing in a single product over a finite time horizon. At the beginning of each period ordering and pricing decisions are made. This means that in each period a different price can be set. For each order made by the retailer there is a fixed ordering cost and variable purchasing cost. Holding cost is incurred for carrying inventory from a period to the next period.

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Furthermore, it is assumed in the paper that demand satisfies the following equation

$$d(p) = \beta p^{-\alpha}, \tag{1}$$

where β is a constant, p is the price and $\alpha > 1$ is the demand elasticity. Finally, it is assumed that price in each period t satisfies $p_{\min} \le p_t \le p_{\max}$. Assuming that all demand has to be satisfied (i.e., loss of demand is not allowed) and using the following notation,

T = model horizon
K = fixed ordering cost
c = per unit purchase cost
h = holding costs per unit per period
p_t = price set in period t
q_t = ordered quantity in period t
I_t = ending inventory in period t,

the problem can be formulated as follows

$$\begin{array}{lll} \max & \sum_{t=1}^{T} d(p_t)p_t - C(D(p)) & \\ \text{s.t.} & p_{\min} \le p_t \le p_{\max} & t = 1, \ldots, T \end{array} \tag{2}$$

where

$$\begin{array}{rlll} C(D(p)) = & \min & \sum_{t=1}^{T} (K\delta(q_t) + cq_t + hI_t) & \\ & \text{s.t.} & I_t = I_{t-1} - d(p_t) + q_t & t = 1, \ldots, T \\ & & q_t, I_t \ge 0 & t = 1, \ldots, T \\ & & I_0 = 0 & \end{array}$$

with

$$\delta(x) = \begin{cases} 0 & for\ x = 0 \\ 1 & for\ x > 0 \end{cases}$$

and $D(p)$ is the demand vector $D(p) = [d(p_1), \ldots, d(p_T)]$.

In problem (2) we maximize the total revenue minus total cost over all periods, such that price is bounded from above and below. If we set $p_{\min} = 0$ and $p_{\max} = \infty$, then price is not restricted in the model. The total cost is represented by $C(D(p))$, which is a 'standard' Wagner–Whitin problem (see Wagner & Whitin, 1958). We minimize ordering, purchasing and holding cost, such that demand is satisfied and order quantity and ending inventory are nonnegative in each period. Furthermore, we may assume without loss of generality that starting inventory is zero.

3. Solution approach

3.1. General solution approach

In this section we propose an exact algorithm that has a running time which is quadratic in the model horizon T. This method was proposed by Thomas (1970) for a similar problem. Thomas considers a more general problem, where the demand functions and the cost parameters may vary over time. The proposed method (in the general case) is explained below. Note that Thomas presented the model as a cost minimization problem, whereas we present it as a profit maximization model.

For $1 \le j \le t \le T$ define p_{jt} as the price vector $p_{jt} = [p_j, \ldots, p_t]$ and define $\pi_{jt}(p_{jt})$ as the total profit if production takes place in period j to satisfy demands in periods $j, \ldots, t$ (we will call this a subplan), i.e.,

$$\pi_{jt}(p_{jt}) = \sum_{k=j}^{t} \left(p_k - c_j - \sum_{i=j}^{k-1} h_i \right) d_k(p_k) - K_j. \tag{3}$$

Furthermore, define π_{jt} as the maximum profit for a subplan consisting of periods $j, \ldots, t$, i.e.,

$$\pi^*_{jt} = \max_{p_{jt}} \pi_{jt}(p_{jt}). \tag{4}$$

Thomas shows that if a setup takes place in period j and the next setup in period t, then the optimal price for period $k = j, \ldots, t-1$ must be set at the value which maximizes

$$\left(p_k - c_j - \sum_{i=j}^{k-1} h_i \right) d_k(p_k).$$

Dependent on the structure of $d_t(p_t)$ we can calculate this optimal price in an analytical way or, if necessary, by a numerical procedure. Substituting the optimal prices in (3) we are able to determine π^*_{jt}. Because it can be shown that the optimal solution consists of a series of consecutive subplans, the following forward recursion enables us to find the optimal profit for the whole model horizon:

$$F(t) = \max_{j=1,\ldots,t} (F(j-1) + \pi^*_{jt}) \text{ for } t = 1, \ldots, T \text{ with } F(0) = 0. \tag{5}$$

3.2. The Bhattacharjee and Ramesh case

For the Bhattacharjee and Ramesh case we can find the optimum of (3) in an analytical way. Substituting demand function (1) and the constant cost parameters (i.e., $K_t = K$, $c_t = c$ and $h_t = h$ for $t = 1, \ldots, T$) in (3) we have that

$$\begin{aligned} \pi_{jt}(p_{jt}) &= \sum_{k=j}^{t} \left[p_k - c - \sum_{i=j}^{k-1} h \right] \beta {p_k}^{-\alpha} - K \\ &= \sum_{k=j}^{t} [p_k - c - (k-j)h] \beta {p_k}^{-\alpha} - K. \end{aligned} \tag{6}$$

Calculating the first order conditions we have for the subplan consisting of periods $i = j, \ldots, t$

$$\frac{\partial \pi_{jt}(p_{jt})}{\partial p_i} = 0 \Leftrightarrow \alpha\beta c {p_i}^{-\alpha-1} + (i-j)h\alpha\beta {p_i}^{-\alpha-1} - (\alpha-1)\beta {p_i}^{-\alpha} = 0$$

or

$$p^*_i = \frac{\alpha(c + (i-j)h)}{\alpha - 1} > 0 \text{ as } \alpha > 1,\ i \ge j \text{ and } c, h \ge 0. \tag{7}$$

Note that p^*_i is not dependent on the other prices set in the periods of the subplan. Furthermore, note that p^*_i does only depend on period j and not on period t, which implies that the optimal price for a single period is only dependent on the starting period of the subplan and independent of the length of the subplan. Finally, one can verify that

$$\left. \frac{\partial \pi_{jt}(p_{jt})}{\partial p_i} \right|_{p_i} > 0 \text{ for } p_i < p^*_i \text{ and } \left. \frac{\partial \pi_{jt}(p_{jt})}{\partial p_i} \right|_{p_i} < 0 \text{ for } p_i > p^*_i,$$

which implies that the maximum profit function for a single period in a subplan is unimodal and that it has a unique optimum at price p^*_i.

If we analyze the second order partial derivative we find

$$\frac{\partial^2 \pi_{jt}(p_{jt})}{\partial {p_i}^2} = -\alpha(\alpha+1)\beta(c + (i-j)h){p_i}^{-\alpha-2} + \alpha(\alpha-1)\beta {p_i}^{-\alpha-1},$$

which is equal to zero for

$$\widehat{p}_i = \frac{(\alpha+1)(c + (i-j)h)}{\alpha - 1} > p^*_i.$$

It is not difficult to verify that the second order partial derivative is smaller than zero for $p_i < \widehat{p}_i$ and larger than zero for $p_i > \widehat{p}_i$. This means that the maximum profit function for a single period in a subplan is concave for $p_i < \widehat{p}_i$ and convex for $p_i > \widehat{p}_i$.

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