ARTICLE IN PRESS
JID: EOR [m5G;January 3, 2017;16:29]
European Journal of Operational Research 000 (2017) 1–11
Contents lists available at ScienceDirect
European Journal of Operational Research
journal homepage: www.elsevier.com/locate/ejor

Interfaces with Other Disciplines

Truthfulness with value-maximizing bidders: On the limits of approximation in combinatorial markets☆

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ARTICLE INFO

Article history:
Received 6 May 2016
Accepted 20 December 2016
Available online xxx

Keywords:
Auctions/bidding
Game theory
Approximation mechanisms

ABSTRACT

In some markets bidders want to maximize value subject to a budget constraint rather than payoff. This is different to the quasilinear utility functions typically assumed in auction theory and leads to different strategies and outcomes. We refer to bidders who maximize value as value bidders. While simple single-object auction formats are truthful, standard multi-object auction formats allow for manipulation. It is straightforward to show that there cannot be a truthful and revenue-maximizing deterministic auction mechanism with value bidders and general valuations. Approximation has been used as remedy to achieve truthfulness on other mechanism design problems, and we study which approximation ratios we can get from truthful mechanisms. We show that the approximation ratio that can be achieved with a deterministic and truthful approximation mechanism with n bidders cannot be higher than $1/n$ for general valuations. For randomized approximation mechanisms there is a framework with a ratio that is tight.

1. Introduction

Auctions have received increasing attention in operations research (Bichler, Schneider, Guler, & Sayal, 2011; Lorentziadis, 2016; Mansouri & Hassini, 2015). In auction theory, bidders are almost always modeled as payoff-maximizing individuals using a quasilinear utility function. Under these utility functions the Vickrey–Clarke–Groves mechanism is the unique mechanism to obtain maximum welfare in dominant strategies (Green & Laffont, 1979). However, there are markets where a pure quasilinear utility function might just not be the right assumption. Sometimes, capacity constraints (Chaturvedi, 2015) or budget constraints (Dobzinski, Lavi, & Nisan, 2012a) need to be considered that has ample effects on the equilibrium bidding strategies and efficiency. Sometimes, however, bidders rather maximize total value subject to a budget constraint.

For example, digital advertising markets have grown substantially in the recent years (Ember, 2015). In *display ad auctions* individual user impressions on a web site are auctioned off. Advertising buyers bid on an impression and, if the bid is won, the buyer's ad is instantly displayed on the publisher's site. Demand-side platforms (DSPs) are intermediaries, who provide the technology to bid for advertisers on such advertising exchanges. A number of papers describe bidding strategies and heuristics in display ad auctions (Chen, Berkhin, Anderson, & Devanur, 2011; Elmeleegy et al., 2013; Feldman, Muthukrishnan, Nikolova, & Pal, 2008; Ghosh, Rubinstein, Vassilvitskii, & Zinkevich, 2009; Zhang, Rong, Wang, Zhu, & Wang, 2016; Zhang, Yuan, & Wang, 2014). Zhang et al. (2014) give an up-to-date overview. In all of these papers the task of the DSP or advertiser is to maximize the values of impressions subject to a budget constraint, which the advertiser set for a campaign.

Value maximization subject to a budget is not limited to display ad auctions. Private individuals often determine a budget before making a purchase, and then buy the best item or set of items (e.g., cars, real-estate) that meets the budget. Actually, in classical micro-economic consumer choice theory, consumers select a package of objects that maximizes value subject to a budget constraint, they do not maximize payoff (Mas-Colell, Whinston, & Green, 1995). Also utility functions used in general equilibrium models such as the one by Fisher or Eisenberg and Gale do not maximize payoff, but the sum of utilities or valuations (Jain & Vazirani, 2010). Maximizing value subject to a budget constraint is also wide-spread in business due to principal-agent relationships (Engelbrecht-Wiggans, 1987). For example, in spectrum auctions, national telecoms have different preferences for different packages of spectrum licenses based on the corresponding net present values of business cases. These billion dollar net present values exceed the financial capabilities of the local telecom by far, but not those of its stakeholder, a multinational, which has mainly

☆ The financial support from the Deutsche Forschungsgemeinschaft (DFG) (BI 1057/1-4) is gratefully acknowledged. A preliminary short version of the paper was presented at the Symposium on Algorithmic Game Theory (Fadaei and Bichler, 2016). We thank the participants for feedback.

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http://dx.doi.org/10.1016/j.ejor.2016.12.031
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long run strategic incentives of operating in the local market. Thus, the stakeholder provides the local telecom with allowances for individual packages based on the underlying net present value. Both parties determine the valuations of different allocations and the principal then sets allowances less or equal to these valuations, which describe how high the agent can bid in order to win a package. Shapiro, Holtz-Eakin, and Bazelon (2013) argue that such predetermined budgets have to do with capital rationing (Paik & Sen, 1995). For agents, these allowances are like sunk costs but they have preferences to win the most valuable packages within budget. Paulsen and Bichler (2016) show that depending on the auction format it can be impossible for a principal to align the incentives of a value-maximizing agents (aka. *value bidders*).

It is straightforward to show that there is no truthful and revenue-maximizing mechanism for value bidders with general valuations or types are impossible. Therefore, we focus on approximation mechanisms for combinatorial auction markets (with a fully expressive XOR bid language). We ask whether there is a truthful mechanism for value bidders, which might not yield the optimal allocation, but an allocation with a good approximation ratio. Combinatorial auction markets allow for general cardinal preferences and can therefore be considered the most general types of market mechanisms, because the preferences are not restricted by the bid language. In such markets, a value bidder has a budget constraint or value describing his maximum willingness-to-pay for each package. The values for a package might include substitutes or complements as in the spectrum auction example, or they might just be additive up to an overall budget constraint as in the display ad example.

1.1. Our results

First, we analyze a truthful Pareto-optimal mechanism for markets with value bidders. We show that such a mechanism exists. Then we study truthful revenue maximizing mechanisms. We focus on revenue rather than welfare. Social welfare is difficult to analyze in environments where bidders have values and budget constraints.[1] We show that for single-minded and single-valued value bidders there are simple truthful mechanisms that maximize revenue, but that this is not possible for multi-minded value bidders. Next, we explore truthful approximation mechanisms.

The need for approximating revenue arises for two reasons. One is because the underlying optimization problem is computationally intractable. This has been the primary motivation for approximation mechanisms as they are described in Nisan (2007). In contrast, we approximate revenue to obtain truthfulness. The approach is similar to that of Procaccia and Tennenholtz (2009): we maximize revenue without considering incentives, and refer to this as optimal revenue. We will then say that a strategy-proof mechanism returns (at least) a ratio α of the optimal if its revenue is always greater than or equal to α times the optimal revenue.

We first look at small markets with two bidders and two items only to get some intuition about possibilities for manipulation, and find out that with a simple assumption there is a truthful deterministic mechanism with a golden approximation factor of $\frac{\sqrt{5}-1}{2}$. A randomized mechanism for the environment with two bidders and two units achieves a factor of $\frac{3}{4}$. Unfortunately, the deterministic mechanism cannot be extended to larger markets.

The analysis of these small markets is valuable on its own right, but it is also helpful for deriving our main result, which says that the best revenue ratio achievable by a deterministic and truthful mechanism with value bidders in a market with n bidders and m homogeneous items is $\frac{1}{n}$, for any $n \geq 2$ and $m > 2$. This can be seen as a negative result, because this ratio can be achieved by selling all objects as a package. The theorem has a straightforward extension to combinatorial markets.

In quasi-linear mechanism design, randomization is often a remedy to achieve higher approximation ratios. Approximation mechanisms for quasi-linear bidders do typically not lead to strategy-proofness with value bidders. However, there is a randomized mechanism by Dobzinski, Nisan, and Schapira (2012b), which is also truthful for value bidders with a simple change of the payment rule. The approximation ratio is tight, and it shows that there exists a gap between the power of randomized and deterministic mechanisms.

1.2. Related literature

Given the substantial literature in social choice, we first position *value bidders* in the literature. The Gibbard–Satterthwaite theorem describes one of the most celebrated results in social choice theory. Gibbard (1973) proved that any non-dictatorial voting scheme with at least three possible outcomes is not strategy-proof. Satterthwaite (1975) showed that if a committee is choosing among at least three alternatives, then every strategy-proof voting procedure is dictatorial. There have been a number of extensions analyzing more specific mechanism design questions for allocation problems without money, typically resulting in impossibility results (Ehlers & Klaus, 2003; Hatfield, 2009; Pápai, 2001).

Quasi-linear preferences are an escape route from the impossibilities found above. The well-known result by Green and Laffont (1979) shows that the VCG mechanism is the unique quasi-linear mechanism, which allows for strategy-proofness and efficiency. There is a huge literature on approximation mechanisms for quasi-linear bidders. The computational hardness for the algorithmic problem of revenue maximization with general valuations in combinatorial auctions is shown to be $O(\sqrt{m})$ (Halldorsson, Kratochvil, & Telle, 2000). This is a natural lower bound on the approximation factor of truthful approximation mechanisms. For quasi-linear bidders randomized approximation mechanisms with the same approximation ratio have been found (Dobzinski et al., 2012b; Lavi & Swamy, 2011). However, the best deterministic truthful approximation guarantee known for general combinatorial auctions is $O(\frac{m}{\sqrt{\log m}})$ (Holzman, Kfir-Dahav, Monderer, & Tennenholtz, 2004).

Closest to our assumptions is the model analyzed by Feldman et al. (2008) in which bidders have an overall budget and a value for ad slots in sponsored search and they want to maximize the number of clicks given their budget. They also argue that a bidder is incentivized to spend the entire budget to maximize exposure or the number of clicks in the market. Feldman et al. (2008) focused on the specifics of ad slot markets with purely additive valuations for homogeneous goods (clicks) and they consider specific scheduling constraints. The overall budget can be seen as the budget for packages in which clicks outnumber a specific threshold. In our model, we do not restrict valuations to be additive and we are interested in packages of heterogeneous items and different budgets for different packages. Recently, we became aware of a working paper by Wilkens, Cavallo, and Niazadeh (2016), who also motivates value maximization, but the paper has a different focus.

1.3. Paper structure

In Section 2 we introduce necessary notation and definitions used throughout the paper. In Section 3 we present a Pareto-optimal mechanism and prove that revenue maximization and

[1] Consider the case of two bidders, one with a high value and low budget for an object, and another one with a lower value and a high budget constraint. In a revenue-maximizing auction, we only need to consider the willingness-to-pay for the object including the available budget and aim for the allocation that maximizes the auctioneers' revenue.

Please cite this article as: S. Fadaei, M. Bichler, Truthfulness with value-maximizing bidders: On the limits of approximation in combinatorial markets, European Journal of Operational Research (2017), http://dx.doi.org/10.1016/j.ejor.2016.12.031