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Innovative Applications of O.R.

Effect of element separation in series-parallel systems exposed to random shocks

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A B S T R A C T

A new general approach to obtaining performance characteristics of complex non-repairable systems in the presence of shocks affecting individual elements and groups of elements is developed. A method of evaluating the effect of elements separation into different groups on the entire system performance is suggested. An important specific case, of the series-parallel systems exposed to internal failures and external shocks is considered. It is shown that in binary systems, separation always improves the system survivability when elements are arranged in parallel and decreases it when elements are arranged in series. A numerical example showing that the separation efficiency can depend on a system performance metric is presented.

1. Introduction

In practice, systems and their components, along with internal failures, are often exposed to external shocks as well, which at many instances provide a suitable model for taking into account the effect of a random environment. Neglecting this influence can lead to errors and misconceptions when analyzing the relevant performance characteristics of complex systems.

Shock models are widely used in different reliability applications (see Cha & Finkelstein, 2011; Cha & Finkelstein, 2016; Finkelstein, 2008; Finkelstein & Cha, 2013; Klefsjo, 1981; Nakagawa, 2007 among others). Traditionally, one distinguishes between two major types of shock models: cumulative shock models (systems fail because of some cumulative effect) and extreme shock models (systems fail due to one single shock). Some generalizations (to name a few) of traditional models have been considered in the literature. For instance, Sumita and Shanthikumar (1985) studied the cumulative shock model when the arrival time and the corresponding shock magnitude are correlated; Gut and Husler (2005) considered the consequences of only the latest shocks; Mallor and Omey (2001) discussed systems that fail when k consecutive shocks with magnitude exceeding a certain level occur; Finkelstein and Cha studied and reviewed numerous shock models in (Finkelstein & Cha, 2013). Most of the shock models in practical reliability (e.g., for modeling maintenance) have been developed under the assumption of the Poisson process of shocks (see, e.g., Chakravarthy, 2012; Huynh et al., 2012; Montoro-Cazorla & Pérez-Ocón, 2011 and references therein). Poisson shock models are usually mathematically tractable and allow for rather compact expressions for the probabilities of interest. Therefore, in this paper, we also assume that the shock processes under consideration follow the Poisson pattern. However, in the future research we plan to consider more general shock processes as well.

One of the effective ways to enhance system survivability is to separate its elements in such a way that an external shock cannot destroy all of them simultaneously. The separation can be performed by spatial dispersion, by encapsulating different elements into different protective casings, etc. At some instances, the separated elements can be exposed to independent random processes of shocks with the same parameters (e.g., as in the case of the spatial separation of electric units in an area characterized by certain lightning frequency (see Fig. 1) when the separated units are exposed to different lightning events, having, however, the same rate). In this case, different shock processes can be considered as statistically identical. On the other hand, these parameters (i.e., the rate and severity) are often different (as, e.g., for the case of thermal shocks affecting separated electronic chips) and can strongly depend on locations after separation.

Some important separation problems have been studied in the literature (Levitin & Hausken, 2010; Levitin & Lisnianski, 2001;

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Nomenclature

$1(\cdot)$	logical function (indicator): 1(TRUE) = 1−1(FALSE) = 1
$E[v]$	expected value of random variable v
N	number of elements in the system
J	number of separated groups in the system
$r(j,i)$	index of ith element belonging to jth group
$s_j(t)$	random performance rate of the separated group j at time t
$S(t)$	random performance rate of a system at time t
$x_i(t)$	binary variable representing state of element i at time t
g_i	nominal performance rate of element i
$G_i(t)$	random performance rate of element i: $G_i(t) = x_i(t)g_i\in (0, g_i)$
$X(t)$	state vector of a group of elements
$\varphi(X(t))$	structure function
$P_j(t,m)$	probability of occurrence of m shocks affecting the jth separated group in [0, t)
$n(j)$	number of elements in jth separated group
K_j	number of possible states of jth separated group
$p_{j,k}(t)$	probability that jth separated group is at state k at time t
λ_j	shock rate for jth separated group
Ω_i, ω_i	parameters of shock survival function for element i
$q_i(m)$	probability that element i survives mth shock
$F_i(t)$	*cdf* of time-to-internal-failure distribution for element i
η_i, β_i	scale and shape parameters of Weibull time-to-failure distribution for element i
d	system demand
$R(t,d)$	probability that system satisfies the demand d at time t
$U(t,d)$	expected cumulative unsupplied demand during the time horizon T

Levitin, Xing, Amari, & Dai, 2014). It was shown that separation is not always beneficial and can decrease the entire system survivability. All these studies assumed a fixed time independent probabilities of external shocks and did not consider shocks as stochastic processes. It was also assumed that the elements affected by the same shock always fail simultaneously, whereas in reality shocks are often not so disastrous and the corresponding failures occur only with certain probabilities.

To the best of our knowledge, this is the first study considering the dynamic effect of external shocks modeled by stochastic point processes on separation efficiency. The presented algorithm provides a novel tool for analysis of dynamic system behavior in the presence of external shocks and internal failures. Our approach also extends the existing models by allowing elements experiencing the same shocks to fail or survive independently. i.e., we assume the corresponding conditional independence meaning that for each shock (given a shock), events of failure (survival) for different component are independent. This assumption is more realistic than that when all effected elements fail or survive simultaneously.

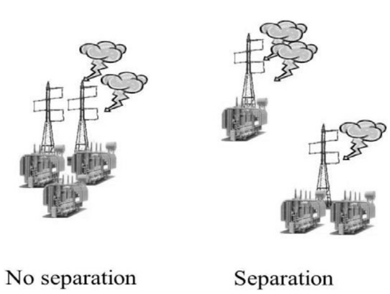

Fig. 1. Example of spatial separation of electric units in an area with lightning.

Section 2 describes the considered model and defines the relevant performance metrics for systems separated into independent groups. Section 3 considers the case of total separation in binary parallel and series systems without internal failures. Section 4 presents an algorithm for evaluating performance metrics for separated series-parallel systems with capacitated binary elements. Section 5 presents numerical examples. Section 6 concludes our work.

2. Problem formulation and general setting

2.1. Structure function of group of elements

Consider a collection of n binary elements composing a group and contributing to functioning of this group. The time dependent random performance rate of the group $s(t)$ can represent its productivity, speed, strength, generation or transmission capacity, processing time etc. depending on specific application. Any element can be either in operating or in failure state and the group performance rate depends on combination of states of its elements.

The state of any element i at time t is represented by a binary random variable $x_i(t)$, where $x_i(t) = 1$ and $x_i(t) = 0$ correspond to operating and failed states of the element, respectively. The state of any group of n elements thus can be represented by the state vector $X(t) = (x_1(t),\ldots,x_n(t))$. The function φ which maps the group state vector $X(t)$ into the group performance rate $s(t)$ is referred to as the group structure function.

If $\varphi(X(t))$ maps the space $\{0,1\}^n$ into {0,1}, it represents the state (operating or failed) of the group at time t. The expectation $E[\varphi(X(t))]$ gives the probability that group is in operating condition at time t (referred to as reliability, availability or survivability depending on a context).

If $\varphi(X(t))$ maps the space $\{0,1\}^n$ into $\boldsymbol{R}$, it represents random group performance rate at time t, whereas $E[\varphi(X(t))]$ defines the expected group performance rate at time t. Consider, for example, a group of n elements operating in parallel. If each element has a performance rate (productivity) g_i in the operational state, the random *cumulative* performance rate of the entire group is $s(t) = \varphi(X(t)) = \sum_{i=1}^{n} x_i(t)g_i$. Alternatively, the structure function can also be defined as the $\boldsymbol{R}^n\to\boldsymbol{R}$ mapping. For the example above, one can define random element performance rate as $G_i(t) = x_i(t)g_i$ and the structure function as $\varphi^*(X(t)) = \sum_{i=1}^{n} G_i(t)$.

The specific type of a structure function as well as the meaning of a performance rate depends on application (see examples in Lisnianski & Levitin, 2003).

2.2. Element separation in binary systems exposed to shocks

We consider the separation of a system consisting of N capacitated binary elements arranged in a series-parallel configuration into J groups consisting of $n(1),\ldots,n(J)$ elements. Each separated group also constitutes a series-parallel subsystem. No element can belong to more than one group.

Each element i is characterized by its nominal performance rate g_i and the *cdf* of time-to-internal-failure distribution $F_i(t)$. Apart from internal failures, the system elements are exposed to random external shock processes. According to the generalized extreme shock model (Cha & Finkelstein, 2011), we assume that any element i survives mth shock with probability $q_i(m)$ and fails with