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Invited Review

The Traveling Purchaser Problem and its variants

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A B S T R A C T

The Traveling Purchaser Problem (TPP) has been one of the most studied generalizations of the Traveling Salesman Problem. In recent decades, the TPP attracted the attention of both researchers in combinatorial optimization and practitioners, thanks to its double nature of procurement and transportation problem. The problem has been used to model several application contexts and is computationally challenging, dealing at the same time with the suppliers selection, the optimization of the purchasing plan and the routing decisions of the purchaser. For the first time after 50 years from its birth, we survey all the research done on the TPP including the most interesting and best performing solution methods proposed so far. We conclude providing some interesting future developments.

1. Introduction

Procurement problems, optimizing costs and revenues for manufacturing companies or firm retailers, have a long history in the specialized literature. The aim of a procurement problem is, in general, to elaborate a purchasing plan that satisfies the demand for a set of products/raw materials while minimizing the procurement costs. Usually, the plan is formalized in terms of two joint decisions, one concerning which suppliers should be selected, and the other one deciding how much should be ordered from each supplier (Aissaoui, Haouari, & Hassini, 2007). This activity is critical in any organization, considering that procurement expenditure typically accounts for a large portion of a firm total cost. For this reason, nowadays, the procurement logistics is still a vivid stream of research (Manerba, 2015).

The study of *routing/transportation problems* optimizing traveling costs dates even back. A routing problem generally aims at finding one or more optimal tours in order to visit a set of geographical locations (customers, suppliers, etc.) from a central depot. The well-known Traveling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP) belong to this category (see Gutin and Punnen, 2002; Toth & Vigo, 2014).

The joint evaluation of both transportation and procurement problems is a more recent stream of research combining the relevant features of the two previous contexts. The *Traveling Purchaser Problem* (TPP) belongs to this stream. In the TPP, given a list specifying products and quantities required, a purchaser has to find a purchasing plan that exactly satisfies the products demand by visiting a subset of suppliers in a unique tour. The objective of the purchaser is to minimize the combined traveling and purchasing cost. In the classical TPP only a single vehicle is involved, even if multi-vehicle variants have already been studied in the literature.

According to Golden, Levy, and Dahl (1981) and Fischetti, Salazar-Gonzalez, and Toth (2007), the TPP represents, together with the family of orienteering problems, one of the most interesting generalizations of the TSP. The large number of papers published on this problem in the last decade demonstrates that it is still attractive for researchers and practitioners. For these reasons, the purpose of the present paper is to survey, for the first time, the existing literature on the TPP. In particular, we will focus on its modeling aspects and solution methods, also including the analysis of its several variants, as the multi-vehicle ones.

The paper is organized as follows. In Section 2, we formally define the TPP, standardize its classifications, point out some interesting properties, and present its many practical applications. In Sections 3 and 4, we survey different Mixed Integer Linear Programming (MILP) formulations for the TPP and the most important polyhedral results, respectively. Sections 5 and 6 present exact and heuristic approaches, respectively. Section 7 analyzes deterministic, dynamic, and stochastic variants of the TPP, as well as its multi-vehicle extensions. Conclusions and open lines of research

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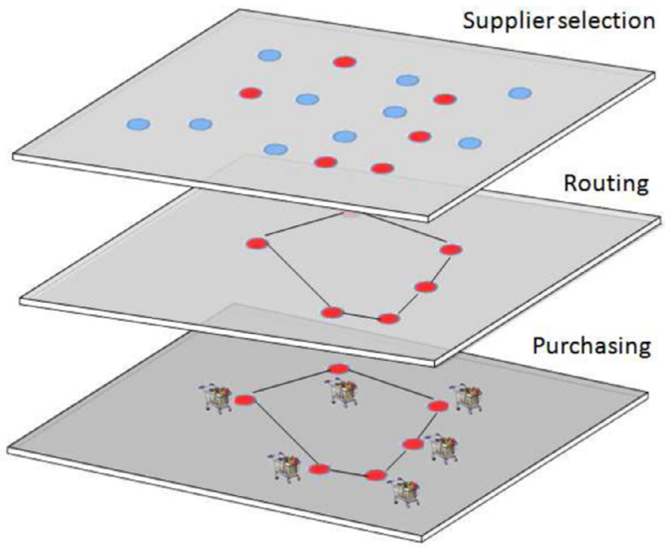

Fig. 1. Components of the TPP in a layered structure.

are drawn in Section 8. Finally, Appendix A presents and compares different sets of benchmark instances.

2. Problem definition and properties

The TPP, in its original form, is a single-vehicle routing and procurement problem defined as follows. Consider a depot 0, a set K of products/items to purchase, and a set M of geographically dispersed suppliers/markets. A discrete demand d_k, specified for each product $k \in K$, can be accomplished in a subset $M_k \subseteq M$ of suppliers at a price $p_{ik} > 0$, $i \in M_k$. Moreover, a product availability $q_{ik} > 0$ is also defined for each product $k \in K$ and each supplier $i \in M_k$. Note that, to guarantee the existence of a feasible purchasing plan with respect to the product demand, the condition $\sum_{i \in M_k} q_{ik} \geq d_k, \forall k \in K$ has to hold. The problem is defined on a complete directed graph $\mathcal{G} = (V, A)$ where $V := M \cup \{0\}$ is the node set, and $A := \{(i, j): i, j \in V, i \neq j\}$ is the arc set. A traveling cost c_{ij} is associated with each arc $(i, j) \in A$. The TPP looks for a simple tour in $\mathcal{G}$ starting and ending at the depot, visiting a subset of suppliers and deciding how much to purchase for each product in each supplier so to satisfy the demand at minimum traveling and purchasing costs.

The great interest in the TPP is probably due to the fact that it challengingly combines supplier selection, routing construction, and product purchase planning. Fig. 1 shows the three components of the problem in a layered framework. It is clear that optimally solving each subproblem separately does not guarantee to achieve the optimal solution for the TPP. The supplier selection (first layer) is a crucial aspect that differentiates TPP from traditional routing problems, linking it to the so-called *routing problems with profits* (Feillet, Dejax, & Gendreau, 2005). Since the main goal of the purchaser is to satisfy products demand, not all the suppliers have to be visited necessarily. In general, the convenience of a visit depends on the trade-off between the additional traveling cost for reaching the supplier and the possible saving obtained by purchasing products at lower prices[1]. The TPP has, in fact, a bi-objective nature, linearly combining in a single objective function the minimization of both traveling and purchasing costs (second and third layer, respectively). This makes the problem of selecting optimal suppliers more complex since the traveling costs optimization pushes the purchaser to select only suppliers that are strictly necessary to satisfy products demand, while the purchasing costs minimization pushes to select a more convenient and potentially larger set of suppliers.

2.1. Common classifications

A first classification comes from the TPP routing nature. As for the TSP, a TPP modeled on a directed graph, where the cost c_{ij} is potentially different from c_{ji}, is named *asymmetric* (ATPP). Otherwise, if $c_{ij} = c_{ji}$ for each arc $(i, j) \in A$, the problem is called *symmetric* TPP (STPP). In the literature, ATPP and STPP are often referred to as *directed* and *undirected* TPP, respectively.

A second common classification concerns the availability of products at the suppliers. If the available quantity of a product $k \in K$ in a supplier $i \in M_k$ is defined as a finite value q_{ik}, potentially smaller than product demand d_k, then the TPP is called *restricted* (R-TPP). The *unrestricted* TPP (U-TPP), instead, considers the case in which supplies are unlimited, i.e., where $q_{ik} \geq d_k$, $k \in K$, $i \in M_k$. Note that U-TPP represents a special case of R-TPP, since having unlimited supplies is equivalent to consider $d_k = 1$ and $q_{ik} = 1$, $\forall k \in K, \forall i \in M_k$. In the literature, several papers refer to R-TPP and U-TPP as *capacitated* and *uncapacitated* TPP, respectively. However, we prefer to adopt the former nomenclature in order to avoid confusion with the concept of vehicle capacity, appearing in the multi-vehicle case. Rarely, these variants are also called *limited-supply* and *unlimited-supply* TPP.

2.2. Complexity

The TPP is $\mathcal{NP}$-hard in the strong sense since it generalizes both the TSP and the Uncapacitated Facility Location Problem (UFLP). This can be proved by the following reductions: (1) the TSP corresponds to an U-TPP where each supplier offers a product that cannot be purchased elsewhere; (2) the UFLP can be seen as an U-TPP where each potential facility location corresponds to a supplier and each customer to a product, $M_k = M$ for all $k \in K$, p_{ik} is the cost of serving customer k from facility i, and $c_{ij} := (b_i + b_j)/2$, $\forall (i, j) \in A$, with b_i the cost of opening facility i.

However, as highlighted by Teeninga and Volgenant (2004), some TPP special cases can be solved trivially, namely (a) when the supplier nearest to the depot sells, for each product, all the required quantity at the lowest price, and (b) when the traveling costs are null. In the latter case, an optimal U-TPP solution can be found by purchasing each product from its cheapest supplier, since any tour connecting these suppliers is optimal (in the R-TPP, instead, the suppliers are first sorted in non-decreasing order of price for each product k, then each product is purchased from its cheapest suppliers in an amount equal to the minimum between the available quantity and the residual demand).

Finally, note that the problem feasibility can be checked polynomially just by inspection of the input data. If a product is not available at any supplier, then no solution exists for the U-TPP. Similarly, for the R-TPP, the infeasibility occurs if it exists a product k such that $\sum_{i \in M_k} q_{ik} < d_k$.

2.3. Applications

In the previous sections, we have presented the TPP as a procurement logistics problem. Interesting enough, its combinatorial structure appears for the first time (in the unrestricted form) in the work by Burstall (1966) to model the scheduling of different jobs on a multi-purpose production line. In that case, products

[1] We remark that suppliers selection in the TPP has to be intended only at an operating level, and depending on the daily product demand, prices, and availabilities. Strategic decisions for selecting the best suppliers based on qualitative criteria (e.g., service quality and reliability) concern another well-studied stream of research (Degraeve, Labro, & Roodhooft, 2000).