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Innovative Applications of O.R.

Optimal setup of a multihead weighing machine

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ABSTRACT

Multihead weighing machines are ubiquitous in industry for fast and accurate packaging of a wide variety of foods and vegetables, small hardware items and office supplies. These machines consist of a system of multiple hoppers that are filled with product which when discharged through a funnel fills a package to a desired weight. Operating the machine requires first to specify the product weight targets or setpoints that each hopper should contain on average in each cycle, which do not need to be identical. The setpoints selection has a major impact on the performance of a multihead weighing machine. Each cycle, the machine fills a package running a built-in knapsack algorithm that opens – or leaves shut – different combinations of hoppers releasing their content such that the total package weight is near to its target, minimizing the amount of product "given away". In this paper, we address the open problem for industry of how to determine the setpoint weights for each of the hoppers *before* starting up the machine, given a desired total package weight. An order statistic formulation based on a characterization of near-optimal solutions is presented. This is shown to be computationally intractable, and a faster heuristic that utilizes a lower bound approximation of the expected smallest order statistic is proposed instead. The solutions obtained with the proposed methods can result in substantial savings for users of multihead weighing machines. Alternatively, the analysis presented could be used by management to justify the acquisition of new machines of this type.

1. Introduction

A multihead weighing machine (hereafter an MWM, sometimes called a combinatorial weighing machine) is a computer-controlled machine used to fill a package with small products or parts with a given target weight. This machine has a wide range of applications in the food industry for packaging pasta, coffee beans, cereals, snacks, candies, vegetables, and even for packing poultry pieces and beef. Its applications cover also the packaging of non-food items, for instance, clips, nails, screws and a variety of other small hardware items. Among the multihead weigher manufacturers, the one with the world leading position has 31,000 MWMs installed all over the world (Ishida Corporation Ltd.). Despite their widespread use, analytical studies aimed at optimally setting up an MWM, a critical step affecting the performance of these machines, are lacking. In this paper, we model and analyze an MWM and propose methods for its optimal setup.

An MWM is composed of a system of feeders, a set of H pool hoppers, a set of H weight hoppers and a discharge chute to the packaging machine (Fig. 1). The product is continuously fed via a central dispersion feeder (usually a vibrating cone) and H radial feeders (vibrating channels) to the pool hoppers. The role of the pool hoppers is to stabilize the product before dropping it into the weight hoppers. The average weight of product μ_i, $i = 1, \ldots, H$, that each hopper should contain must be specified by an operator before starting the machine. These average weights need not be identical. Once the machine is started, each cycle a built-in knapsack-like algorithm selects a subset of hoppers whose sum of observed weights is closest to the target value (for recent discussions on the knapsack problem see Schauer, 2016 and Wishon & Villalobos, 2016). Next, a computer opens the selected hoppers releasing the product through the discharge chute into the package. Some hoppers can therefore remain shut filled with product from cycle to cycle. One cycle is repeated for each package. The performance of an MWM heavily depends on the initial hopper weights $\{\mu_i\}$. In industrial practice, operators currently use trial and error rules to setup the hopper weights based on the product to pack and the target weight of the package, but such setting-up operation may be far from optimal. In this paper, we focus on the

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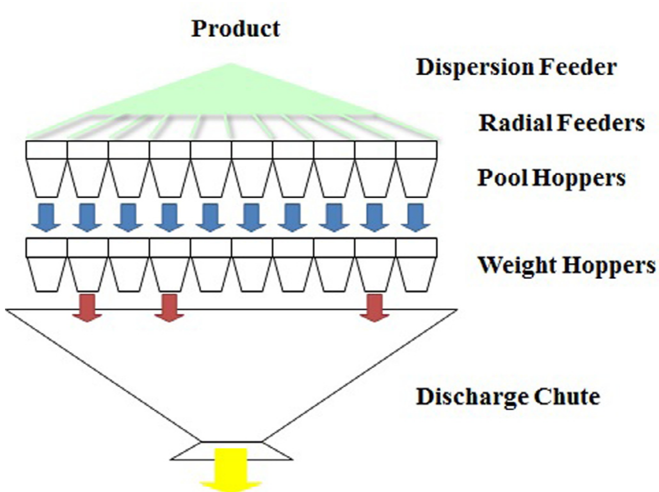

Fig. 1. A single-layered multihead weighing machine. The central dispersion cone is connected to a series of vibrating radial feeders, one per hopper, which can be controlled individually providing individual controllability to each hopper mean weight setpoint μ_j.

analysis and optimal setup of MWMs with a single layer of hoppers (Fig. 1), the most common type of MWM in industrial use.

Practically all of the extant technical literature related to MWMs (see, e.g., James & Storer, 2005, Kameoka, Nakatani, & Inui, 2000, Karuno, Nagamochi, & Wang, 2007, Imahori, Karuno, Nagamochi, & Wang, 2011; Karuno, Nagamochi, & Wang, 2010), which mostly originates in Japan where MWMs were first developed, deals with the repetitive problem of finding the best combination of hoppers to open in each cycle, proposing different versions of Knapsack formulations, but does not address the *setup* problem of selecting the hopper weights *before* starting up the machine. The MWM problem we address below is somewhat related to canning and process mean targeting problems (Arcelus & Rahim, 1996; Goethals & Cho, 2011; Pollock & Golhar, 1998; Raza & Turiac, 2016; Selim & Al-Zu'bi, 2011) but they differ in that in the latter there is no selection combination difficulties involved.

MWM's are based on an empirically observed "variance reduction" technique: it was noted that by filling a package from the combination of product from several hoppers, negative correlations are induced between the weights of product in opened hoppers given that they are random variables that are selected in each cycle subject to a constraint in their sum (which gives the package total weight) (Kameoka et al., 2000). The negative correlations reduce the mean square error of the packages weight, "giving away" less product while satisfying the target constraint.

The rest of the paper is organized as follows. The next section presents a mathematical formulation of the MWM setup problem and an exact approach for simple problems (i.e. when only few combinations of hoppers opening are considered). Next, the behavior of good solutions obtained by numerical search is characterized. These characteristics are then used in Section 4 to develop a heuristic approach to the optimal MWM setup problem. The paper ends with recommendations and directions for further research.

2. Formulation of the optimal setup problem of a multihead weigher machine

Let w_j be the observed weight of the product contained in the jth hopper in a particular cycle of operation, $j = 1, 2, \dots, H$ where H is the number of hoppers in the machine. Each cycle the machine fills up a package with product released from a subset of the hoppers and the depleted hoppers are refilled. Assume w_j is a realization of the random weight $W_j \sim N(\mu_j, \sigma_j^2 = \alpha^2\mu_j^2)$, $\mu_j > 0$, $j = 1, 2, \dots, H$ and assume each weight is independent of other weights $W_i (i \neq j)$. The proportionality constant α (with $\alpha < 1$) is assumed known and given as it depends on the product to be packed. The proportional relation between mean and standard deviation of the weights is known to exist in this type of machines (e.g., see Kameoka et al., 2000). We point out that as long as $\sigma = f(\mu)$ holds for any known f, the methods developed below also apply after trivial modification. However, given that the available empirical evidence (see, e.g. Beretta, 2010, p. 87) indicates that a simple linear relation of the form $\sigma = \alpha\mu$ fits the weight data very well, it was adopted in what follows.

A *setup* of the machine consists of specifying the values of the setpoints $\boldsymbol{\mu}' = (\mu_1, \mu_2, \dots, \mu_H)$, to which, according to our assumption, also determine the hopper weight variances σ_j^2, $j = 1, \dots, H$, for a given target value T that specifies the minimum weight content of each package to be filled. Once the machine is setup, the combinatorial weigher machine starts to fill packages of product, solving a knapsack algorithm per package. Our goal is to determine the best setpoints $\boldsymbol{\mu}$ according to some specific criteria on the weight content of the packages.

While there are different knapsack formulations that have been reported in the MWM literature, most of them utilize a linear objective function and linear constraints. In this section, we assume the machine has a built-in algorithm that solves for each package the deterministic knapsack problem:

$$\min \quad \omega = \sum_{j=1}^{H} \delta_j w_j \quad \text{subject to:} \quad \omega = \sum_{j=1}^{H} \delta_j w_j \geq T, \tag{1}$$

where δ_j is either 0 or 1. In this formulation, the total observed package weight ω is required to be as small as possible but larger or equal to the given target package weight *T*. Prior to observing the hopper weights $\{W_j = w_j\}$ in any cycle, the total package weight $\mathcal{W}$ is the minimum of K dependent, not identical normal random variables X_i for $i = 1, 2, \dots, K$, subject to the constraint $\mathcal{W} \geq T$, where K equals the total number of possible combinations of opened/closed hoppers from which the knapsack algorithm can select (choosing the δ_j variables above). We hasten to point out that *we are not concerned with solving the knapsack problem; the knapsack problem is internal to the machine and considered given*. We are concerned with determining the setpoints of the machine, i.e., the mean weights in each hopper, which are the "inputs" of the system as depicted in Fig. 2.

The optimal setpoints could be found from the distribution of the optimum objective function value (i.e., the package weight $\mathcal{W}$) of a random Knapsack where W_j substitutes w_j in (1). However, there are only limited results related to this distribution (Prekopa, 1995, p. 526). They are asymptotic results as the number of hoppers $H \to \infty$ under the assumption the hopper weights W_i's are $U(0, 1)$ random variables, which is clearly not our case. Also related is work on the average performance of greedy algorithms for stochastic knapsack problems (Diubin & Korbut, 2003; Mastin & Jaillet, 2015). Greedy algorithms, however, are not implemented in an MWM given that these machines are aimed at high volume production and even minor savings per cycle (due to slightly better knapsack solutions) represent substantial savings over the life of the MWM. Furthermore, this thread of work also considers asymptotic analyses for $H \to \infty$ assuming the W_i's have support in [0,1]. In the remainder of this section, we describe how to compute the exact *moments* of the total weight package $\mathcal{W}$ in problem (1) and how this leads very rapidly to computational complexities in practice.

If all possible combinations of any number of hoppers can be selected to open (or close) in a cycle, then clearly there are