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Decision Support

Deriving rankings from incomplete preference information: A comparison of different approaches

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A R T I C L E I N F O

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Volume-based methods for decision making under incomplete information like the SMAA family of methods provide rich probabilistic information to support decision making. However, they usually do not directly generate a unique ranking of alternatives. Methods to create such a unique ranking from incomplete preference information typically select one parameter vector, either by mathematical programming approaches or by averaging, and then apply a preference model using this parameter vector. In the present paper, we develop several models to infer a complete ranking or a complete preorder of alternatives directly from the probabilistic information provided by volume-based methods without singling out a specific parameter vector. We compare the results obtained by these models to those obtained with a single parameter approach in a computational study. Results indicate small, but significant differences in the performance of methods, as well as in the probability that additional preference information might worsen, rather than improve, the results.

1. Introduction

During the last decades, many methods of decision making under incomplete information have been developed to overcome the difficulties which decision makers have in exactly specifying their preferences. For a seminal review of early developments in this field, see e.g., Weber (1987), for more recent developments e.g., Ehrgott, Figueira, and Greco (2010). In Section 2, we will provide a brief review of relevant literature. Even with incomplete preference information, it can be necessary to ultimately recommend a clear choice, or indicate a complete and unambiguous ranking of alternatives. Methods for decision making under incomplete information thus have to bridge the gap from incomplete preference information to a complete ranking of alternatives.

Not all methods for decision making under incomplete information aim at providing such unambiguous results. Some provide only incomplete, but robust relations. Volume-based approaches, which form an important stream of decision making under incomplete information, provide probabilistic rather than crisp information about rankings. One of the best known methods in this stream is the SMAA (Stochastic Multiobjective Acceptability Analysis) family of methods (Lahdelma, Hokkanen, & Salminen, 1998; Lahdelma & Salminen, 2001; Tervonen & Figueira, 2008). The main results of these methods consist in different indices which describe probabilities that an alternative occupies a certain rank, or that one alternative is considered to be better than another one. In the present paper, we study the question of how these indices can be used to derive an unambiguous ranking of alternatives. We develop several models for that purpose, and compare them in a computational study to direct parameter estimation from incomplete information. In the present paper, we focus on multicriteria decision problems under certainty, but conceptually the approach could also be extended to decisions under risk.

The remainder of the paper is structured as follows: In section two, we give a brief review of previous literature related to our problem. In section three, we develop the models to obtain a unique ranking based on the probabilistic information obtained from SMAA and similar methods. In section four, we present a computational study to evaluate our models. Results of the study are contained in section five, section six discusses these results and provides some outlook onto future research.

2. Preference models for incomplete information

Many approaches to multicriteria decision analysis employ some preference parameters such as attribute weights, partial utility values, or threshold levels, and assume that these parameters can be specified exactly. Approaches for decision making under incomplete information question this assumption and argue that decision makers might not be able to provide exact parameter values and analysts might not be able to elicit exact values from them.

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Incomplete information on parameters can take many forms. Some approaches use intervals, or ordinal statements about rankings of attribute weights (Park & Kim, 1997; Sarabando & Dias, 2010). Other approaches, which are often labeled as preference disaggregation techniques (Jacquet-Lagrèze & Siskos, 2001), use comparisons between alternatives to establish constraints on preference parameters. In general, any information can be interpreted as a set of constraints which define a set of *admissible* parameter vectors. In the following, we will refer only to weights as preference parameters, but most approaches can easily be applied also to other types of parameters.

The vast literature on decision making under incomplete information can roughly be classified into three streams. One stream is interested in deriving robust conclusions, which are compatible with the preference information available. These methods were originally developed in the area of decision making under risk with unknown probabilities (Kmietowicz & Pearman, 1984), and were soon after adapted to uncertain weights in decision problems with multiple criteria (Hazen, 1986; Kirkwood & Sarin, 1985). These models were later on extended to incomplete information on values (Park, Kim, & Yoon, 1996; Park & Kim, 1997; Park, Lee, Eum, & Park, 2001) and criteria hierarchies (Salo & Hämäläinen, 1995). More recently, Robust Ordinal Regression (ROR) methods (Greco, Mousseau, & Słowiński, 2008) have followed a similar approach.

All these methods provide some relations between the alternatives. In the terminology of ROR (Greco et al., 2008; Kadziński & Tervonen, 2013), these relations are denoted as necessary and possible preference relations. Necessary preference, sometimes also called dominance (Hazen, 1986; Park & Kim, 1997), is established between two alternatives A_i and A_j, if alternative A_i is preferred to A_j for all admissible weight vectors. The necessary preference relation usually is not a complete order relation on the set of alternatives, since pairs of alternatives might exist for which preference is possible in both directions. Possible preference between two alternatives A_i and A_j is established, if there exists at least one admissible weight vector for which A_i is preferred to A_j. Possible preference is mostly not an asymmetric relation, any asymmetric element of the possible preference relation is also an element of the necessary preference relation. As the relations are incomplete, these methods cannot be used to establish a complete order relation between alternatives.

The second stream of literature tries to identify one particular weight vector in the set of admissible vectors, which is considered to represent (or to approximate) the “true” weights of the decision maker. The origins of this stream can be traced back to early methods for estimating attribute weights from pairwise comparisons of alternatives using optimization models (Srinivasan & Shocker, 1973). A particularly well known method in this stream is the UTA method (Jacquet-Lagrèze & Siskos, 1982). Later on, many extensions to this method were developed (e.g., Beuthe & Scannella, 2001; Bous, Fortemps, Glineur, & Pirlot, 2010). In the context of more recent ROR methods, the concept of representative value functions (Kadziński, Greco, & Słowiński, 2012) is based on a similar approach. Since these methods apply a standard preference model, they generate a complete order of alternatives, as long as the model used has this property.

The third stream of literature considers the entire set of admissible weights, and compares the volumes of different subsets of this set to derive (usually probabilistic) statements about the ranking of alternatives. This concept goes back to the domain criterion of Starr (1962), which was applied to the context of multicriteria decisions by Charnetski and Soland (1978) and Eiselt and Laporte (1992). A similar approach was later on provided in the VIP software (Dias & Clímaco, 2000). Perhaps the most widely known method in this stream is SMAA (Stochastic Multiobjective Acceptability Analysis) developed by Lahdelma et al. (1998) and Lahdelma and Salminen (2001).

The original SMAA method defined rank acceptability indices r_{ik}, which indicate the probability that alternative A_i is ranked on position k. Later on, Leskinen, Viitanen, Kangas, and Kangas (2006) introduced the concept of pairwise winning indices p_{ij}, which indicate the probability that alternative A_i is preferred to A_j. Both sets of indices are usually determined using Monte-Carlo simulation, in which different admissible weight vectors are sampled, the resulting evaluations of alternatives are computed, and the indices are calculated as the fraction of admissible weights for which the respective properties hold. Since the precision of simulation is naturally limited by the number of parameter vectors generated, recent literature (Kadziński & Tervonen, 2013) proposes to complement SMAA with optimization models to obtain exact results.

Since the SMAA method generates probabilistic information, it cannot directly be used to obtain a complete order relation. However, as SMAA generates a representative sample of the set of admissible weight vectors, it is possible to determine a central parameter vector within this set, which can then be used to rank alternatives (Lahdelma et al., 1998). Leskinen et al. (2006) discuss how different voting rules can be used to derive a cardinal score for alternatives from pairwise winning indices.

3. Volume-based ranking models

We consider a decision problem which involves N_{alt} alternatives A_i evaluated according to N_{crit} criteria, thus $A_i = (a_{i1}, \dots, a_{iN_{crit}})$. We denote the rank acceptability indices by r_{ik} and pairwise winning indices by p_{ij}. We first formulate models to derive strict preference relations, and then extend them to take indifference into account.

Finding an order of alternatives from rank acceptability indices corresponds to a standard assignment problem (Wagner, 1975) of alternatives to ranks. We introduce a binary variable x_{ik} indicating that alternative A_i is assigned to rank k. The standard formulation of an assignment problem (Wagner, 1975, p. 184) uses the constraints

$$\sum_{k=1}^{N_{alt}} x_{ik} = 1 \quad \forall i = 1, \dots, N_{alt} \tag{1}$$

and

$$\sum_{i=1}^{N_{alt}} x_{ik} = 1 \quad \forall k = 1, \dots, N_{alt} \tag{2}$$

indicating that each alternative is assigned to one rank (1) and that exactly one alternative must be assigned to each rank (2).

Different objective functions could be used in this problem. A natural objective is to maximize the average probability of the assigned ranks, i.e.,

$$\max f_{Sum} = \sum_{i=1}^{N_{alt}} \sum_{k=1}^{N_{alt}} r_{ik} x_{ik} \tag{3}$$

Since a total of N_{alt} assignments are made, $\sum_i \sum_k x_{ik} = N_{alt}$ is constant and (3) will maximize the average probability. It is also possible to consider the joint probability of the entire ranking. Under the assumption that all assignments are stochastically independent (which is, however, violated due to the constraints), this probability could be approximated by

$$\max f_{Prod} = \prod_{i,k:x_{ik}=1} r_{ik} \tag{4}$$

To calculate the joint probability of an ordering of alternatives exactly would require all the conditional probabilities that some