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Decision Support

Winner determination in geometrical combinatorial auctions

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ABSTRACT

We consider auctions of items that can be arranged in rows. Examples of such a setting appear in allocating pieces of land for real estate development, or seats in a theater or stadium. The objective is, given bids on subsets of items, to find a subset of bids that maximizes auction revenue (often referred to as the *winner determination problem*). We describe a dynamic programing algorithm which, for a *k*-row problem with *connected* and *gap-free* bids, solves the winner determination problem in polynomial time. We study the complexity for bids in a grid, complementing known results in literature. Additionally, we study variants of the geometrical winner determination setting. We provide a NP-hardness proof for the 2-row setting with gap-free bids. Finally, we extend this dynamic programing algorithm to solve the case where bidders submit connected, but not necessarily gap-free bids in a 2-row and a 3-row problem.

1. Introduction

In combinatorial auctions, bidders can place bids on combinations of items, called packages or bundles. Clearly, combinatorial auctions allow bidders to better express their preferences compared to the traditional auction formats, where bidders place bids on individual items. In particular, it makes sense to use a combinatorial auction when complementarities or substitution effects exist between different items.

Research on combinatorial auctions was triggered by applications such as the FCC spectrum auction (Jackson, 1976) and auctions for airport time slots (Rassenti, Smith, & Bulfin, 1982). For an introduction to combinatorial auctions, we refer to the book edited by Cramton, Shoham, and Steinberg (2006); for a survey of the literature, we refer to Abrache, Crainic, Gendreau, and Rekik (2007) and de Vries and Vohra (2003).

One important challenge within this domain is, given the bids, to decide which items should be allocated to which bidder, i.e., which bids to accept. In general, this *winner determination problem* is NP-hard (Van Hoesel & Müller, 2001), and does not allow good approximation results (Sandholm, 2002).

We discuss a combinatorial auction in a restricted topology. In this setting, an item corresponds to a rectangle, and all items are arranged in (a limited number of) rows, see Fig. 1 for an example. Notice that the individual items (or rectangles) need not have the same size. A bid consists of a set of items satisfying some restrictions (see Section 2 for a precise problem definition), together with a value. The objective is to select a set of bids that maximizes the sum of the expressed values, while making sure that each item is present at most once in a selected bid.

There are several situations in practice that motivate this specific geometric setting. We mention the following:

- Real estate. Goossens, Onderstal, Pijnacker, and Spieksma (2014) describe how space in a newly erected building, to be used for housing and commercial purposes, is allocated using a combinatorial auction. The geometric structure of each of the levels of the building features the properties described here. Quan (1994) reports on empirical studies in real estate auctions. Several of these studies have focused on verifying and quantifying the *afternoon effect*. This afternoon effect describes similar items consistently selling for significantly less in later rounds in multi-object sequential auctions. Quan (1994) even reports on finding this effect in a large real estate auction (122 lots) of vacant lots that are geographically similar. The lots were formed in 23 groups based on their geographical proximity. In 20 out of the 23 groups of properties, the afternoon effect was present with the last bidder paying on average one-third less than the first bidder for geographically similar lots. A combinatorial auction, by selling all items simultaneously, can mitigate this effect.
- Mineral rights. Imagine a region that is partitioned into lots, with the lots organized in rows. For sale is the right to extract minerals, oil or gas found on or below the surface of the lot.

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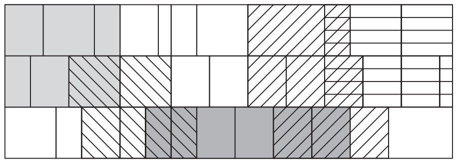

Fig. 1. An example of an instance with 3 rows and 5 bids.

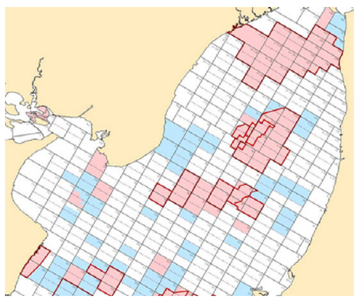

Fig. 2. Oil and gas leases managed by the Texas General Land Office. Taken from: http://www.glo.texas.gov/GLO/agency-administration/gis/gis-data.html.

Clearly, having adjacent lots allows for exploration and production efficiencies, a complementarity. For more about this particular setting, we refer to Cramton (2007). Fig. 2 shows an example of oil and gas leases neatly arranged in rows.

- Seats in a grandstand, theater or stadium. In some of these cases, one can even assume that a grid, consisting of rows and columns, is given where each cell represents a seat. Typically, demand exists for sets of adjacent seats – think of a family of four going to a ball game, or a group of friends visiting a concert. The complementarities that people perceive from adjacent seats offer possibilities for combinatorial auctions. Although tickets are usually sold at a fixed price, there are occasions where sports teams have auctioned off (part of) their seat licenses[1]. Another, not unrealistic, example is the selling of airline tickets[2].
- Laboratory experiments. Scheffel, Pikovsky, Bichler, and Guler (2011) provide results of laboratory experiments testing different auction formats in five different value models. Their third value model has six pieces of land arranged in two rows on a shoreline. Bidders are interested in bundles that contain at least one lot at the shore. Their fourth value model has nine pieces of land arranged in three rows. In Scheffel, Ziegler, and Bichler (2012) a local synergy value model is used in which 18 items are arranged rectangularly in three rows with bidders interested in adjacent items. Kazumori (2010) ran experiments using 16 items arranged rectangularly in four rows. Each agent has a base value for each item and a varying level of additional interest for adjacent items. These laboratory experiments required solving very small instances of the winner determination problem. In case one were to increase the number of pieces of land, or one wants to run a continuous auction, or one wants to give bidders all sorts of feedback, an efficient algorithm for the winner determination becomes a necessity.

In all these cases, it is clear that complementarities between adjacent items exist; a combinatorial auction is best-placed to take these effects into account.

The main goal of this paper is to show how the specific geometric setting described above can be used to efficiently solve the winner determination problem (which is hard in general), using dynamic programing procedures. Additionally, we settle the complexity of the winner determination problem for bidding in a grid. This paper does not address mechanism design or bidding strategy issues.

Goossens et al. (2014) show that when a constraint is imposed stating that a bidder can have at most one winning bid, the winner determination problem is NP-hard even if all items are arranged on a single row. Hence, to have any prospect of coming up with a positive result, we allow bidders to win multiple bids. Notice however that, under some conditions on the bids, an optimal solution where each bidder has at most one winning bid is guaranteed to exist. This is the case, for instance, if the bids placed by each bidder satisfy at least one of the following conditions:

- every pair of bids of a bidder has a non-empty intersection;
- all bids from the same bidder are *super-additive*, i.e. for any two disjoint sets S and T it should hold that the bid expressed on $S \cup T$ is at least as large as the sum of the expressed bids on S and T.

The first condition is satisfied if bidders place only one bid. Bids coming from (truthful) *single-minded* bidders, who are only interested in a specific set of items or a superset of these items, also satisfy the first condition. Indeed, more formally, single-minded bidders have a set of items S^* and a value v^* such that their valuation $v(S) = v^*$ for all $S \supseteq S^*$, and $v(S) = 0$ for all other S (see Nisan, Roughgarden, Tardos, & Vazirani, 2007). The second condition corresponds to the bids that can be expressed using a bidding language consisting of OR-bids (see Nisan, 2000). Summarizing, in these cases, our dynamic program will result in an optimal solution where each bidder has at most one winning bid.

1.1. Literature

Our problem is a special case of finding a maximum-weight independent set in a geometric intersection graph. In such a graph, there is a node for each bid (in our case: a (connected) set of rectangles), and two nodes are connected if and only if the corresponding bids overlap. Finding a maximum-weight independent set in a geometric intersection graph is a well-studied problem for several types of intersection graphs. For instance, in the work of Rothkopf, Pekeč, and Harstad (1998), it is shown that if all items are arranged in a single row, and bids are only allowed for subsets of consecutive items, the resulting winner determination problem is polynomially solvable. These results follow from the equivalence of this problem to finding a maximum-weight independent set in an interval graph. For an overview on results for more general intersection graphs we refer to Chan and Har-Peled (2012). Depending upon particular properties of the geometric figures, different complexity results are known. We restrict ourselves here to mentioning that for fat objects (like squares and disks) polynomial time approximation schemes are known (see Erlebach, Jansen, & Seidel, 2001; Hochbaum & Maass, 1985). The important special case of finding a maximum-weight independent set in a rectangle intersection graph is considered in Chalermsook and Chuzhoy (2009).

In the context of auctions, Babaioff and Blumrosen (2008) and Christodoulou, Elbassioni, and Fouz (2010) study mechanism designs for the setting where geometric figures in the plane are the objects for sale. They sketch applications in advertising, renting land for exhibitions and licenses for location-based services. They show how to guarantee a certain fraction of the optimal welfare for

[1] For instance, the New York Jets (NFL) have earned over 16 million dollars in an online auction for seat licenses. See http://www.nfl.com/news/story/09000d5d80c071a4/article/jets-earn-more-than-16-million-in-online-psl-auction.

[2] For instance, the article found at the following URL describes how some carriers require persons whose weight exceeds a given number to buy two (adjacent) tickets: http://www.cheapair.com/blog/travel-tips/airline-policies-for-overweight-passengers-traveling-this-summer/.