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The Undirected Capacitated General Routing Problem with Profits

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ABSTRACT

In this paper we introduce and study the *Undirected Capacitated General Routing Problem with Profits* (UCGRPP). This problem is defined on an undirected graph where a subset of vertices and edges correspond to customers, which are associated with a given profit and demand. The profit of each customer can be collected at most once. A fleet of homogeneous capacitated vehicles is given to serve the customers. The objective is to find the vehicle routes that maximize the difference between the total collected profit and the traveling cost in such a way that the demand collected by each vehicle does not exceed the capacity and the total duration of each route is not greater than a maximum given time limit. We propose a mathematical formulation of the problem and introduce valid inequalities to strengthen the corresponding continuous relaxation. Moreover, we provide an aggregate formulation that allows us to introduce further inequalities. Then, we propose a two–phase exact algorithm for the solution of the UCGRPP. In the first phase, a branch-and-cut algorithm is used to solve the aggregate formulation and to identify a cut pool of aggregate valid inequalities to be used in the second phase, where a branch-and-cut algorithm is implemented to optimally solve the UCGRPP. Computational results on a large set of problem instances show that the use of the aggregate formulation is effective, making the two-phase exact algorithm able to optimally solve a large number of instances.

1. Introduction

The class of *routing problems with profits* is becoming more and more popular as witnessed by the increasing number of contributions in the last years. This is mainly due to the variety of practical applications which can be modeled as one of the numerous variants of routing problems with profits, but also to the intrinsic scientific challenge raised by these problems which strongly differ in nature from their inherited parents, i.e., the classical routing problems. In fact, contrary to what happens in classical routing where all customers must be served, in routing problems with profits a crucial decision is the identification of the subset of “convenient” customers to serve.

Routing problems with profits are traditionally classified in two main classes: *Node routing problems with profits*, where customers are located on the vertices of a graph, and *arc routing problems with profits*, where customers correspond to arcs/edges of a graph. The literature on *node routing problems with profits*, which are simply identified by the scientific community as *vehicle routing problems with profits*, is much wider than the one on *arc routing problems with profits*. We refer the reader to Archetti, Speranza, and Vigo (2014a) for a recent survey on *vehicle routing problems with profits* and to Archetti and Speranza (2014) for a survey on *arc routing problems with profits*. A further classification can be made on the basis of the objective function and the problem constraints. Although there is a high variety of different problem settings, we can distinguish the following three main classes:

1. *Orienteering:* The objective function is the maximization of the total profit collected. In this case, a maximum duration constraint or a capacity constraint (or both) are imposed on vehicle routes. The most studied problem of this class is the *Orienteering Problem*, where customers are located on vertices of the graph and a single vehicle with a maximum duration constraint is available. It arises in several real-world contexts. A first application, described by Tsiligirides (1984), refers to the case of a travelling salesman with not enough time to visit all possible customers. The extension to the case of multiple vehicles is the *Team Orienteering Problem*. For a survey, see Vansteenwegen, Souffriau, and Van Oudheusden (2011). In the class of *arc routing problems with profits* we have the *Arc Orienteering Problem*

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(see Souffriau, Vansteenwegen, Vanden Berghe, & Van Oudheusden, 2011), for the single vehicle case, and the *Team Orienteering Arc Routing Problem* (see Archetti, Corberán, Plana, Sanchis, & Speranza, 2014b) for the multiple vehicle case.

2. *Prize-Collecting:* The objective function is the minimization of the total traveling cost. In this case a constraint establishing to collect a minimum amount of profit is imposed. The main problem studied in this class is the *Prize-Collecting Traveling Salesman Problem*, where customers are located on vertices and a single vehicle is available (see Feillet, Dejax, & Gendreau, 2005a). It has several practical applications. For instance, Lopez, Carter, and Gendreau (1998) describe a problem that arises in the steel industry, when scheduling steel coil production, which can be interpreted as a generalization of the Prize-Collecting Traveling Salesman Problem.
3. *Profitable Tour:* The objective function is the maximization of the difference between the total collected profit and the traveling cost. Side constraints can be defined on vehicle routes on the basis of the application. In the case of customers located on vertices, this problem is known as the *Profitable Tour Problem* for the single vehicle case (see Feillet et al., 2005a) and as the *Capacitated Profitable Tour Problem* for the multiple vehicle case (see Archetti, Feillet, Hertz, & Speranza, 2009). When customers are located on arcs/edges, we have the *Prize-Collecting Rural Postman Problem* for the single vehicle case (see Aráoz, Fernández, & Zoltan, 2006) and the *Profitable Arc Tour Problem* for the multiple vehicle case. There are a lot of applications of these problems, especially in logistic distribution and transportation. For instance, applications of the profitable arc tour problem related to the domain of the tactical freight transportation-planning problem in the car industry are described in Feillet, Dejax, and Gendreau (2005b).

For an exhaustive literature review on *routing problems with profits* the reader is refereed to Archetti et al. (2014a) and Archetti and Speranza (2014). There exist classes of routing problems of different nature (i.e., without profits) concerning the selection of elements to be visited. We mention, for instance, the *Traveling Purchaser Problem* (see Angelelli, Mansini, and Vindigni, 2016; Bianchessi, Mansini, and Speranza, 2014; Infante, Paletta, and Vocaturo, 2009; Riera-Ledesma and Salazar-González, 2012 and Gendreau, Manerba, and Mansini, 2016 for some recent references).

In this paper, we consider the *general routing problems with profits*, in which customers are located both on some vertices and some edges of the graph. Overall, general routing problems have aroused a growing interest in the last decade. Several authors have highlighted that the transformation of these problems in equivalent ones is not effective in most cases. One problem that has received more attention is the *Mixed Capacitated General Routing Problem*, that is the problem in which the demand of a set of customers located in vertices and arcs/edges of a mixed graph must be serviced by a fleet of homogeneous vehicles. Recent contributions concerning this problem are those related to lower bounding procedures (see Bach, Hasle, & Wøhlk, 2013), tailored exact algorithms (see Bosco, Laganà, Musmanno, and Vocaturo, 2013 and Irnich, Laganà, Schlebusch, & Vocaturo, 2015), and non-exact approaches (see Bosco, Laganà, Musmanno, & Vocaturo, 2014). Beraldi, Bruni, Laganà, and Musmanno (2015) have studied the stochastic counterpart of the problem. To the best of our knowledge, the only problem studied in the literature dealing with profits either distributed along edges or concentrated on vertices of a logistic network is the *Bus Touring Problem* (BTP) presented in Deitch and Ladany (2000). In the BTP, a single vehicle is available and the total profit collected is maximized while some side constraints, such as maximum route duration or cost, have to be satisfied.

We introduce and study the *Undirected Capacitated General Routing Problem with Profits* (UCGRPP), which is defined on an undirected graph and customers are located both on some vertices and some edges of the graph. A profit and a demand are associated with each customer. A fleet of homogeneous capacitated vehicles is available to serve customers. Each vehicle has a capacity and a maximum route duration constraint. The objective is to choose a subset of customers to serve and define the routes visiting those customers such that the difference between the total collected profit and the traveling cost is maximized.

The UCGRPP generalizes many single-vehicle and multiple-vehicle node, arc, and general routing problems with profits. Therefore, it underlies several applications in contexts where calling on specific entities or traversing streets is not mandatory but implies a reward. An example is the route-planning problem for people interested in visiting several attractive points of a new destination. The attractiveness is determined by the tourist sites that are visited and by the scenic road segments that are traveled. Sites (vertices) and road segments (edges) are weighted with a non-negative value (profit) which denotes the amount of pleasure from visiting or traversing them for the first time, respectively. The aim is to maximize the attractiveness or, as our view, the difference between the total attractiveness and the traveling cost. The number of routes that must be generated depends on the period of stay. In the manuscript of Deitch and Ladany (2000), a single route is defined. In fact, the BTP refers to the one-period case which generally corresponds to one day. On the contrary, the multiple-period case can be modeled through the UCGRPP. An additional example is related to the case of collection of recycling goods whenever some customers may be represented as individual vertices and groups of customers as edges, depending on their demand and dispersion. In particular, in a reverse logistics system where not all customers must be served, there are some big centers that produce a large amount of these goods and other small centers that can be considered as a single producer. In general, the UCGRPP arises in contexts where arc routing problems with profits are defined. Nevertheless, in many cases, modeling a routing problem with profits through data on vertices and data on edges allows to better exploit its structure.

This paper makes the following contributions to the literature: (*i*) we introduce a new problem (UCGRPP) and provide two mathematical programs; (*ii*) we adapt from the literature theoretical and methodological results in order to solve it effectively; (*iii*) we provide new problem instances and computational results. Our general routing problem is transformed into an arc routing problem with profits for which we present an aggregate formulation, i.e., a model that uses variables aggregating the number of traversals performed by all vehicles for every edge. It is similar to the one presented in Belenguer and Benavent (2003) for the *Capacitated Arc Routing Problem* (CARP) and is able to provide a good upper bound in a reasonable computing time. In addition, the proposed aggregate formulation allows us to introduce valid inequalities for the UCGRPP that are used to reduce the optimality gap. We propose an exact method for solving instances in the field of the general routing problem with profits. Specifically, we implement a two–phase algorithm for the solution of the UCGRPP. In the first phase, a branch-and-cut algorithm (see Mitchell, 2002 for an introduction) is used to solve the aggregate formulation and to identify a cut pool of aggregate valid inequalities to use in the second phase. In the second phase, a branch-and-cut algorithm is implemented to optimally solve the UCGRPP.

The paper is organized as follows. In Section 2, the UCGRPP is formally described, a property of optimal solutions is proved, a mathematical formulation is provided and valid inequalities are introduced. In Section 3, the aggregate formulation is provided and the aggregate valid inequalities are introduced. In Section 4, the