European Journal of Operational Research 000 (2016) 1–15

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

The within groups and the between groups Myerson values

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ARTICLE INFO

Article history:
Received 8 February 2016
Accepted 1 August 2016
Available online xxx

Keywords:
Myerson value
Communication centrality
Betweenness centrality
Social networks

ABSTRACT

In this paper we revisit the additive decomposition that Gómez et al. (2003) introduced for the Myerson value of a symmetric game when viewed as a centrality measure. First, we generalize this decomposition, extending it to general games. This approach permits us to look at the Myerson value of a player as a certain modulus of a two component vector. One of them, the within groups Myerson value, determines which part corresponds to the profit from the coalitions that a given player is in, whereas the other, the between groups Myerson value, evaluates the opportunities that player has as intermediary in the communication among others. These two values are then characterized using additivity and other properties related with previous interpretation: (A) The competitive advantages (or disadvantages) of a null player in a game with restrictions given by a graph (measured in terms of his Myerson value) are due to his ability to intermediate among the others. (B) In the same context, those players essential to coalitions that generate worth cannot obtain profit by intermediating. When restricted to certain symmetric games, the corresponding values can be considered as centrality measures, as they satisfy natural properties that reinforce this interpretation.

1. Introduction

An n-person cooperative game or a TU game models a conflict in which several actors (players) can obtain payoffs by cooperating. The outcome of a given coalition depends on its members. Implicitly, it is assumed that all players can communicate with each other and also that all possible coalitions are feasible. Myerson (1977) modified this last assumption by introducing restrictions in the communication among players through a network, mathematically represented by a graph. In this new setting, some of the coalitions become infeasible. Myerson, then, defined the graph-restricted game, a new TU game in which the outcome of a coalition is the sum of the payoffs obtained in the original game by its maximally connected (in the graph) subcoalitions. Next, Myerson proposed the Shapley value (Shapley, 1953) of this new game as a point solution for games with such restrictions. Moreover, he gave a characterization of the defined value, now called the Myerson value, using two properties: efficiency in connected components and fairness. Later, Myerson (1980) gave another characterization of this value, replacing fairness by the balanced contributions property.

From its origin the Myerson value has received much attention and has been generalized to related frameworks. Winter (1992) pointed out that the Myerson value admits a potential, following the approach suggested by Hart and Mas-Colell (1989). Van den Nouweland, Borm, and Tijs (1992) extend the Myerson value to the case in which the communication possibilities of the agents are modeled by a hypergraph. Jackson and Wolinsky (1996) introduced the equal bargaining power rule, an extension of the Myerson value for network games. Algaba, Bilbao, Borm, and López (2001) characterized the Myerson value for union stable structures. Calvo, Lasaga, and van den Nouweland (1999) extend it in a natural way to the case of games with probabilistic graphs, in which each pair of nodes has a given probability of direct communication, these probabilities being independent. Gómez, González-Arangüena, Manuel, and Owen (2008) consider a more general setting, in which a probability distribution over the set of all possible graphs is given. Casajus (2009) introduced the graph-χ-value, an outside-option-sensitive extension of the Myerson value. Béal, Remilá, and Solal (2010) study cooperative games with a tree on the set of players representing the limited cooperation possibilities and introduce natural extensions of the average rooted-tree solution, first developed in Herings, van der Laan, and Talman (2008). Recently, González-Arangüena, Manuel, and del Pozo (2015) obtain

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http://dx.doi.org/10.1016/j.ejor.2016.08.003
0377-2217/

another extension of the Myerson value for the case of weighted-link graph restricted games.

Gómez et al. (2003) proposed using the Myerson value as a centrality measure for actors in a social network. Social networks analysis studies the consequences of the restrictions of the different actors in their communications and then in their opportunities of establishing (potentially) profitable relations. Among other goals, network analysis tries to obtain indices, as objective as possible, to measure not directly observable (or hypothetic) variables such as influence, opportunities or better position. Following Wasserman and Faust (1994) an actor in a social network will be *prominent* if his ties make him particularly visible to other actors in the network. This prominence should be measured considering not only the adjacent ties, but also indirect paths involving intermediaries. *Centrality* is one of the two aspects of prominence.[1] Those actors with the most access or the most control will be the more central in the network.

Social networks researchers have defined during the past decades a number of centrality measures, reflecting different facets of the concept. Freeman (1977, 1979) distinguishes three of these centrality measures:

- *Degree centrality*, that identifies centrality of a node with its degree, that is, the number of links incident on it.
- *Closeness centrality*, that represents *independence*, that is, the possibility of communicating with many others depending on a minimum number of intermediaries.
- *Betweenness centrality*, that focuses on the communication *control*, that is, the possibility to intermediate in the communication of others.

In all the three cases the hub in a star is the node with the most privileged position from a relational point of view. An actor in this position: can communicate directly with all the others, and he then has maximal degree; is maximally close to the others, thus having the highest closeness centrality, and is essential for the communication of all subsets of actors-nodes not including him, thus reaching the highest betweenness centrality.

In Gómez et al. (2003), authors assumed that actors in a network are simultaneously players in a TU game which model their economic interests. In their model an actor's centrality is computed as his Myerson value in the given situation (the game plus the network), that is, his Shapley value in the graph-restricted game. To avoid differences among actors due to the game and not to their relative positions in the graph, authors restrict themselves to symmetric games, that equally treat all the players. Given a social network, each specific choice of the game determines a centrality measure (not only in a cardinal sense; order of the actors centralities can swap). Authors therefore claim they define a family of centrality measures. Below, they define the *communication centrality* of a node i as the portion of its total centrality corresponding to the payoffs received from coalitions including it as a member, and its *betweenness centrality* as the payoffs that node i obtains from coalitions in which it is not a member but is (or can be) useful to connect them.

In the first part of this paper, we revisit the idea of the additive decomposition of the Myerson value, first proposed in Gómez et al. (2003) only for the class of symmetric games, extending their definition to the whole class of TU games. One of the defined components corresponds to the profit that a player obtains from the coalitions that he is in and we will call it the *within groups* Myerson value (from now on, WG-Myerson value). The other captures which part of the Myerson value is the profit that a given player obtains from the (disconnected) coalitions that he is not in, but plays a role in connecting, and we will call this part the *between groups* Myerson value (from now on, BG-Myerson value). Immediately after we explore the properties of each one of the two summands and we characterize them. Both of these values satisfy, as the Myerson value itself does, independence of the remaining components. Unlike the Myerson value, the WG-Myerson value satisfies extended link monotonicity, or generalized stability, in the sense that, in the case of convex games, adding a link to the graph does not reduce the value of any node-player. On the other hand, the BG-Myerson value, as the Myerson value itself, satisfies link monotonicity or stability in the case of superadditive games (adding a link to the graph does not reduce the value of both incident nodes). Also both values are characterized in a parallel manner. The WG-Myerson value is the unique additive point solution for games with cooperation restricted by a graph that satisfies the null player property and coincides with the Myerson value for essential players in the game (that is, players such that coalitions not containing them are worthless[2]). The BG-Myerson value is the unique additive point solution for games with cooperation restricted by a graph that assigns zero to essential players and coincides with the Myerson value for null players. If the difference between the Myerson value and the Shapley one is interpreted as an individual social capital index as in González-Arangüena, Khamelnitskaya, Manuel, and del Pozo (2011), a consequence of previous characterizations is that all the social capital of a null player is due to his ability to join groups to which he does not belong, whereas the social capital of essential players is entirely due to their communication abilities.

The Myerson value can then be considered as the additive composition of two allocation rules respectively measuring within and between groups communication abilities in the network. From this point of view, and to some extent, this paper provides a justification of the Myerson value in terms of centrality.

The rest of the paper is devoted, considering once again the more restricted class of the symmetric TU games, to exploring the behavior and to establishing some properties of the two measures defined as additive components of each centrality measure defined in Gómez et al. (2003). In particular, the communication centrality is the same for symmetrical players in the graph; for a convex game, it is maximal for players connected with all the others (for example, the hub of a star) and it is minimal for isolated nodes; in a chain (if the game is convex) it does not decrease from the end nodes to the median one(s) and, of course, the measure inherits the other cited properties of the WG-Myerson value. Similarly, the betweenness centrality of Gómez et al. (2003) also satisfies symmetry in the graph; it is maximal in the hub of a star (superadditivity is needed), it is minimal for isolated nodes and for nodes with only one incident link (again under superadditivity) and, if the game is convex, in a chain, it does not decrease from the end nodes to the median one(s). The other mentioned properties of the BG-Myerson value are also satisfied by its restriction to symmetric games.

The paper is organized as follows. After this introduction, a section of preliminaries is placed. Section 3 is devoted to definitions, to the proof of some properties and to characterizing the two values. In Section 4, the restriction of the WG-Myerson value to symmetric games is used as a communication centrality measure for social networks, and several properties that reinforce this interpretation are proved. Similarly, for the corresponding restriction of the BG-Myerson value. Immediately after, both measures are calculated for a general symmetric game and several relevant networks and games, comparing the figures obtained with those corresponding to Freeman's closeness and betweenness centrality measures.

[1] The other is *prestige*, a concept relevant only in the case of directional relations (Wasserman & Faust, 1994).

[2] In the case of simple games, such a player is called a veto player.