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Innovative Applications of O.R.

Evasive flow capture: A multi-period stochastic facility location problem with independent demand

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ABSTRACT

We introduce the problem of locating facilities over a finite time horizon with the goal of intercepting stochastic traffic flows that exhibit evasive behavior, which arises when locating weigh-in-motion systems, tollbooths, vehicle inspection stations, or other fixed flow-capturing facilities used for law enforcement. The problem can be formulated as a multi-stage, mixed-integer stochastic program; however, under certain independence assumptions, this can be reformulated as a large two-stage stochastic program, enabling us to solve much larger instances. We additionally propose an algorithm based on Lagrangian relaxation that separates the reformulated stochastic program into a variant of a deterministic knapsack problem and a sum of time-decoupled single-period stochastic programs that can be solved independently. The model and algorithm are tested on instances involving road networks of Nevada and Vermont. A comparison with the previously studied single-period stochastic programming approach shows that the newly proposed multi-period model substantially reduces the expected cost.

1. Introduction

We consider a class of flow-capturing problems (FCPs) where the objective is to locate facilities in order to intercept flow-based customers traveling between their origin and destination nodes (Berman, Larson, & Fouska, 1992; Hodgson, 1990). Some applications of the FCP and its variants include the optimal location of discretionary facilities such as convenience stores, gas stations and bank automatic teller machines (Berman, Bertsimas, & Larson, 1995; Hodgson, Rosing, Leontien, & Storrier, 1996a; Wu & Lin, 2003), billboards (Averbakh & Berman, 1996), vehicle inspection stations (Gendreau, Laporte, & Parent, 2000; Hodgson, Rosing, & Zhang, 1996b; Mirchandani, Rebello, & Agnetis, 1995), weigh-in-motion systems (AlGadhi, 2002; Šelmić, Bešinović, & Teodorović, 2011), traffic counting points (Yang & Zhou, 1998), rail park-and-ride facilities (Horner & Groves, 2007), and alternative-fuel stations (Capar, Kuby, Leon, & Tsai, 2013; Kim & Kuby, 2012; Kuby & Lim, 2005; 2007; Kuby et al., 2009; Lim & Kuby, 2010; MirHassani & Ebrazi, 2012). More recently, Marković, Ryzhov, and Schonfeld (2015) proposed a variant of the FCP in which traffic flows exhibit non-cooperative behavior by changing their travel paths to avoid fixed facilities. This is of paramount importance when locating flow-capturing facilities that are used for law enforcement (e.g., weigh-in-motion systems, inspection stations and safety checkpoints). In the evasive flow-capturing problem (EFCP), a flow is considered captured only if every origin-destination path in a pre-specified set (typically the set of k shortest paths) is covered by a facility. Otherwise, the flow travels along the shortest path in the set that is not covered by a facility. The EFCP has a substantially different structure from the standard FCP, and cannot be solved using the same techniques.

In this paper we extend the concept of EFCP by allowing facilities to be allocated at *multiple time points* during the planning horizon. Since the new model accounts for the time component, it can better address situations when traffic flows change over time. This ability is highly relevant to practice, since traffic flows experience steady growth (Fig. 1). Moreover, an agency deploying facilities may not have sufficient funds to implement all the required facilities at once; such agencies are often restricted with their annual budget and thus may have to stretch their investments over several years. This issue is particularly important in EFCP, where several expensive facilities are often needed in order to intercept even a single non-cooperative flow. The lifespan of these facilities may also be limited, giving rise to the problem of optimally timing their deployment. These issues motivate our study of multi-period EFCP, where facilities can be allocated at multiple time points during the planning horizon.

We make the following contributions:

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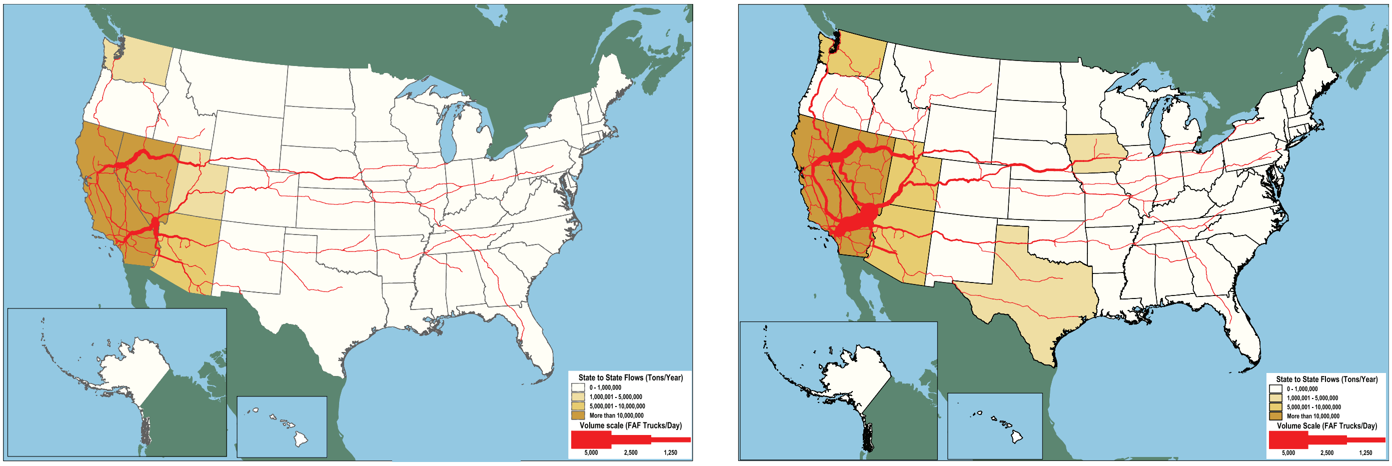

(a) Truck flows in 2010

(b) Truck flows in 2040

Fig. 1. Current and estimated truck traffic flow intensities in Nevada (FHWA, 2011).

1. We formulate the multi-period stochastic EFCP as a multi-stage, mixed-integer stochastic program (SP) where intensities of flows and their willingness to avoid facilities are represented by discrete random variables. To our knowledge, this is the first model to consider a multi-period stochastic flow-capturing problem.
2. We prove that, under independence assumptions, our multi-stage stochastic SP is reduced to a large two-stage SP. With additional modeling improvements that allow partial linear relaxation without affecting the optimal solution, we are able to optimally solve realistically sized instances (e.g., problems on the road network of Nevada) with mathematical programming software.
3. For larger instances, we propose an algorithm, based on Lagrangian relaxation, that separates the multi-period problem into a variant of a deterministic knapsack problem and a sum of time-decoupled single-period SPs that can be solved independently. This essentially decomposes the multi-period stochastic EFCP into a set of comparatively easier subproblems. We also propose a heuristic for finding a good initial upper bound for the Lagrangian method.
4. We implement the model and algorithm on problems involving the road networks of Nevada and Vermont. We obtain insights into the behavior of optimal deployment strategies in the multi-period problem, and demonstrate that the multi-period model adds value over the single-period model. For a relatively modest increase in traffic flows (3.2% annually), the multi-period model provides additional cost savings of 3–4%, which translates to a significant dollar value in practice. Much greater savings are observed with higher increases in traffic flows.

The paper is organized as follows. Section 2 reviews the literature on FCP and multi-period facility location problem. Section 3 formulates the multi-period stochastic EFCP, while Section 4 proposes a Lagrangian heuristic to tackle this problem. Section 5 provides numerical examples where we illustrate application of our model on the road network of Nevada. Section 6 computes the savings that we achieve by applying the multi-period approach. Section 7 compares the performance of the proposed Lagrangian heuristic against simple application of a mathematical programming software. Section 8 draws conclusions and states possible extensions.

2. Literature review

Many flow-capturing models were proposed since the FCP was first introduced. We have already mentioned various applications that were considered in the literature, while a detailed description of about 30 different FCPs is provided in Zeng, Castillo, and Hodgson (2010). The first paper to address multi-period location of flow-capturing facilities is a recent work by Chung and Kwon (2015), which is concerned with allocation of electric car charging stations. (A variant of the same problem was addressed in Li, Huang, and Mason, 2016.) This work, like ours, motivates the need for a multi-period approach by limited budgets for facility deployment and nonstationary demand for facilities. However, to our knowledge, our work is the first to consider a stochastic variant of the multi-period FCP. We also provide additional modeling flexibility that allows the decision-maker to accumulate the budget over time, as our case study shows, this ability plays a crucial role in the structure of the optimal solution.

In traditional facility location problems, multi-period variants have been studied extensively since the pioneering work of Ballou (1968) and Wesolowsky (1973). Some of this work includes Galvao and Santibanez-Gonzalez (1992) and Drezner (1995) for a *p*-median problem, Hinojosa, Puerto, and Fernández (2000) and Melo, Nickel, and Da Gama (2006) for multi-commodity capacitated facility location, Albareda-Sambola, Fernández, Hinojosa, and Puerto (2009) for an incremental service facility location problem, Chardaire, Sutter, and Costa (1996) and Contreras, Cordeau, and Laporte (2011) for the uncapacitated facility location problem, and Jena, Cordeau, and Gendron (2015) for the location problem with modular capacity changes. On the other hand, literature on multi-period *stochastic* facility location is relatively sparse. The single-facility case was studied by Berman and Odoni (1982) and by Carson and Batta (1990), while Jornsten and Bjorndal (1994) considered multiple facilities. More recent work has addressed uncertain demand (Aghezzaf, 2005; Hernández et al., 2012), uncertain interest rates as well as demand (Nickel, Saldanha-da Gama, & Ziegler, 2012), and uncertain costs and requirements along the planning horizon (Albareda-Sambola, Alonso-Ayuso, Escudero, Fernández, & Pizarro, 2013). For an overview of the facility location literature we refer the reader to Owen and Daskin (1998), while a review focusing specifically on the stochastic variants of the problem is provided in Snyder (2006). Table 1 shows the size of largest numerical instances considered in recent publications, which

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