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Mitigating variance amplification under stochastic lead-time: The proportional control approach

Xun Wang, Stephen M. Disney*

Logistics Systems Dynamics Group, Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, UK

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ABSTRACT

Logistic volatility is a significant contributor to supply chain inefficiency. In this paper we investigate the amplification of order and inventory fluctuations in a state-space supply chain model with stochastic lead-time, general auto-correlated demand and a proportional order-up-to replenishment policy. We identify the exact distribution functions of the orders and the inventory levels. We give conditions for the ability of proportional control mechanism to simultaneously reduce inventory and order variances. For AR(2) and ARMA(1,1) demand, we show that both variances can be lowered together under the proportional order-up-to policy. Simulation with real demand and lead-time data also confirms a cost benefit exists.

1. Introduction

We investigate the performance of the order-up-to (OUT) and proportional order-up-to (POUT) inventory control policies via the variance of the inventory and orders under a stochastic lead-time. Variability in inventory systems is commonly generated by uncertainties in demand, supply, transportation, and manufacturing. This variability can be amplified by poorly designed replenishment policies (Chen, Dresner, Ryan, & Simchi-Levi, 2000; Lee, Padmanabhan, & Whang, 1997). Fluctuations in the replenishment orders and inventory levels pose an operational threat to companies. High order variance (a.k.a. the bullwhip effect) brings uncertainty to the upstream supplier, and reduces supply chain efficiency. Similarly, high inventory variance results in high safety stock levels and/or poor customer service, which in turn leads to inflated inventory cost.

Logistics uncertainty and stochastic shipping delays are a major component of supply chain risk. In recent years, production and distribution systems have become increasingly global, exposing supply chains to more volatility than ever before. Global transportation routes, involving air, truck, rail and ocean freight modes, have long and variable lead-times, due to external factors such as seasonality effects, security and customs delays and slow steaming. Uncertain lead-times sometimes trigger another effect called *order crossover*, where replenishments are received in a different sequence than they were ordered. Whilst these two concepts do not necessarily imply each other (Riezebos, 2006; Zipkin, 1986), a highly variable lead-time often results in order crossover. This is especially so in global supply chains where container liners may take different routes, overtake each other at sea, and stop at different ports along the way. Furthermore, individual containers may be held up for customs inspections at national borders.

Hayya, Bagchi, Kim, and Sun (2008) classified the research on stochastic lead-time into three schools: the Hadley–Whitin School (Hadley & Whitin, 1963), which assumes that the probability of order crossover is so small that it can be totally ignored; the Zipkin–Song School (Song, 1994; Zipkin, 1986), which assumes that goods are processed sequentially (perhaps in some sort of first-in-first-out queue) so that order crossover cannot happen; and the Zalkind School (Bradley & Robinson, 2005; Robinson, Bradley, & Thomas, 2001; Zalkind, 1978), which takes order crossover into account and discovers that inventory cost and safety stock can be reduced by considering this effect. Models of this kind are first introduced by Finch (1961) and Agin (1966), which gave the correct expression for the distribution of the number of outstanding orders. Zalkind (1978) determined the optimal target inventory level to minimize total cost. Bagchi, Hayya, and Chu (1986) showed the importance of considering order crossover when setting safety stock. Robinson et al. (2001) highlighted that order crossover has a significant impact on inventory control and should not be ignored. The aims of these studies are either to derive (approximate) relevant distributions or to decide safety stock parameters.

* Corresponding author. Tel.: +44 2920876310.
E-mail addresses: WangX46@cardiff.ac.uk (X. Wang), DisneySM@cardiff.ac.uk (S.M. Disney).

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In recent years the impact of order cross-over on inventory management is gaining academic attention. Chatfield, Kim, Harrison, and Hayya (2004) and Kim, Chatfield, Harrison, and Hayya (2006) have investigated the bullwhip effect with stochastic lead-time, adopting the assumptions of i.i.d. demand and the OUT replenishment policy. Hayya, Harrison, and Chatfield (2008) considered the inventory cost optimization problem under order crossover using regression on empirical data. Hayya, Harrison, and He (2011) further studied the impact of order-crossover on inventory cost, assuming deterministic demand and exponentially distributed lead-time. Bischak, Robb, Silver, and J.D. (2014) showed that taking into account order crossover and using an approximate *effective lead-time* deviation allows companies to reduce inventory costs.

Another stream of research has shown that the POUT policy is effective at smoothing the bullwhip effect at a cost of increased inventory variability (Chen & Disney, 2007; Gaalman, 2006). However, most studies on bullwhip effect require at least a predictable, if not a constant, lead-time; while existing research on stochastic lead-time problems do not explicitly tackle the amplification problem. Disney, Maltz, Wang, and Warburton (2016) tried to fill this gap by considering an inventory system with stochastic lead-time and order-crossover. They derived the distribution of orders and inventory under stochastic lead-time and discussed the impact of the proportional OUT policy on costs and safety stocks. However, the demand pattern is restricted to i.i.d. and no formal proof is given for the superiority of the POUT policy over the OUT policy.

This paper is a sequel to Disney et al. (2016) in which we extend, sharpen and refine their results in the following ways: (1) we identify the distributions of order and inventory under stochastic lead-time and auto-correlated demand; (2) we provide conditions when the OUT and POUT policies minimizes inventory variability under the ARMA(p, q) demand process; (3) we examine the possibility of simultaneous reduction of inventory and order variances by proportional control. Below we list our contributions in more detail.

- We develop a state space approach which allows us to derive the probability density functions of orders and inventory under the POUT policy, arbitrarily distributed stochastic lead-time and general ARMA(p, q) demand. The pdfs then allows us to derive exact expressions for the inventory and order variances.
- We give a necessary condition for when the OUT policy minimizes the inventory variance under general ARMA demand and a stochastic lead-time. Based on this condition we prove that the OUT policy is never optimal for minimizing inventory variance when order crossover is present and demand is temporally independent.
- We give a precise condition under which the inventory and order variances can be reduced simultaneously by optimizing the proportional controller in the POUT policy. Parametrical combinations for this condition are derived for special cases of AR(2) and ARMA(1,1) demand. Simultaneous reduction of inventory and order variance via proportional control is possible for the majority of demand processes.

The paper is organized as follows. In Section 2 we introduce notation and modeling basics. Section 3 contains the main results, which includes an exact approach to obtain the distribution of order and inventory, conditions for the optimality of the OUT policy, and conditions for the simultaneous improvement of inventory and order variances. In Section 4 we numerically investigate the impact of demand correlation and lead-time uncertainty. The cost implications of the proportional policy are also provided based on real demand and lead-time data. Finally we conclude and discuss our results in Section 5. Proofs that are not outlined in the main text are presented in the Appendix A.

Table 1
Commonly used notation in this paper.

Variables (time-dependent random processes)	
y_t, z_t	Variables in the ARMA model
ε_t	Gaussian i.i.d. variable with zero mean and unit variance
d_t	Demand
$\hat{d}_t$	Demand forecast
$\hat{D}_t$	Lead-time demand forecast
S_t	Order-up-to level
o_t	Order
i_t	Net inventory level
w_t	Work-in-process
IP_t	Inventory position, $IP_t = i_t + w_t$
ξ_t	Pipeline Status
$x_t(\xi)$	Sub-process of $\{x_t\}$ with pipeline status ξ
Parameters (constant)	
ϕ, θ	Auto-correlation and moving average parameters in the ARMA demand model
ss	Safety stock level
$1-\lambda$	Proportional feedback controller
L^+, L^-	Maximum and minimum lead-time
h, b	Unit holding and backlog cost
First-order moment	
$\mathrm{E}(x)$, μ_x	Expectation of x
$\mathrm{E}(x;\ \xi)$	Expectation of $x_t(\xi)$
Second-order moments	
$\Sigma_{xy}(\tau)$	Mutual covariance function between x and y with time difference τ
$\Sigma_{xx}(0)$, Σ_{xx}	Autocovariance matrix of x
$\Sigma_{xy}(0;\ \xi)$	Mutual covariance between $x_t(\xi)$ and $y_t(\xi)$
Probabilities and distribution functions	
p_L	Probability of lead-time being L periods long
$\psi_x(\cdot)$	Probability density function of x
$\Psi_x(\cdot)$	Cumulative distribution function of x
$\bar{\Psi}_x(\cdot)$	Complementary cumulative distribution function of x
$\varphi(x\|\mu, \sigma^2)$	Probability density function of normal distributed variable x with mean μ and variance σ^2
Matrices	
I	Appropriately dimensioned identity matrix
$\mathbf{1}$	Appropriately dimensioned unit column vector
A^T	Transpose of A
A^{-1}	Inverse of A
diag$\{\cdots\}$	Block-diagonal matrix

2. Modeling the demand and ordering policy

In this section we establish the model including the objective function, demand process, forecasting and inventory control policies, sequence of events, and the balance equations. We focus on a periodic review inventory system where the system states are defined on $\mathbb{R}$. The lead-time, defined on $\mathbb{N}^+$, is a positive random variable following any arbitrary non-negative discrete distribution that is independent over time. The assumption of discrete lead-time is natural as the lead-time is measured in units of the review period in periodic systems (Disney et al., 2016). Since our model allows for order crossovers, there are no restrictions on the lead-times of consecutive orders. The demand is a normally distributed ARMA(p, q) process. Both the lead-time distribution and demand correlation are known in advance. In practice this knowledge can be realized by statistically analyzing historical data.

Table 1 lists commonly used notation. Importantly we denote $\Sigma_{xy}(\tau)$ as the mutual covariance function between the random variables x and y, with time difference τ. $\mathrm{E}(x)$ or μ_x is the expectation of x. Variables x and y don't have to be scalars. If x and y are vectors and each contains m and n scalar random variables respectively, $\Sigma_{xy}(\tau)$ is an $m \times n$ matrix and $\mathrm{E}(x)$ is a $1 \times m$ vector. When $\tau = 0$, $\Sigma_{xx}(0)$ is the autocovariance matrix of x. Sometimes we write this as Σ_{xx} if no other confusion would occur. The