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Discrete Optimization

The Hierarchical Mixed Rural Postman Problem: Polyhedral analysis and a branch-and-cut algorithm

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ABSTRACT

The Hierarchical Mixed Rural Postman Problem is defined on a mixed graph where arcs and edges that require a service are divided into clusters that have to be serviced in a hierarchical order. The problem generalizes the Mixed Rural Postman Problem and thus is NP-hard. In this paper, we provide a polyhedral analysis of the problem and propose a branch-and-cut algorithm for its solution based on the introduced classes of valid inequalities. Extensive computational experiments are reported on benchmark instances. The exact approach allows to find the optimal solutions in less than 1 hour for instances with up to 999 vertices, 2678 links, and five clusters.

1. Introduction

Arc routing problems differ from vehicle (node) routing problems in the fact that they require some links (arcs and/or edges) to be traversed (serviced) instead of nodes to be visited. We refer the reader to the recently published book by Corberán and Laporte (2014) for more information on arc routing problems. Some of the most challenging problems in this research field take into account the presence of priority levels assigned to links, thus dividing them into clusters and defining an order in which clusters have to be serviced. The introduction of priority levels characterizes the Hierarchical arc routing problems, which arise naturally in different application contexts: from snow plowing where main streets have to be cleaned before secondary streets and residential ones (see Alfa and Liu, 1988; Chernak, Kustiner, and Phillips, 1990; Haslam and Wright, 1991; Lemieux and Gampagna, 1984), to garbage collection (see Bodin & Kursh, 1978).

In all cited applications the underlying problem is a Hierarchical Chinese Postman Problem (HCPP) that requires finding a minimum cost tour starting and ending at the depot, and traversing all the edges in the specified hierarchical order. Dror, Stern, and Trudeau (1987) first introduced the HCPP and the concept of hierarchy in the service of a cluster of streets. They also proved that the HCPP is NP-hard but can be solved in polynomial time when the graph is directed or undirected, the order relation between the clusters of arcs and edges is complete, and each cluster induces a connected graph, and proposed a $O(p|V|^5)$ time algorithm to solve this special case, where p is the number of priority levels and $|V|$ is the number of vertices. Ghiani and Improta (2000) showed that this particular case of the HCPP can be solved as a matching problem on an auxiliary graph with at most $p|V|$ vertices. Later on, Korteweg and Volgenant (2006) proposed an exact algorithm to solve the same problem in $O(p|V|^4)$.

Letchford and Eglese (1998) studied a slightly related problem, the Rural Postman Problem with Deadline Classes (RPPDC). It is a variant of the undirected RPP where the edges requiring service (required edges) are divided into classes and a deadline is given for the completion of the service in each class. The objective of the problem is to find a route starting and ending at the depot while traversing the required edges before their deadline. They proposed a branch-and-cut algorithm that was tested on instances with one and two deadline classes and with up to 50 vertices and 110 edges. Although similar in some aspects, the RPPDC differs from the HMRPP in three main features. In the HMRPP a link of the priority class k is not allowed to be serviced before all the links in the previous classes have already been serviced, while this is possible in the RPPDC. The objective functions are different, since in the HMRPP three different costs are associated with the traversal of a link (before it has been serviced, while servicing it, and after it has been serviced), while in the RPPDC each edge has only one traversal cost. Finally, the problem studied in Letchford and Eglese (1998) is defined on an undirected graph while the problem we study in this paper is defined on a mixed graph.

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Gélinas (1992) proposed a dynamic programming algorithm to solve the problem on a directed graph, whereas Cabral, Gendreau, Ghiani, and Laporte (2004) discussed how two natural objectives can be considered for a hierarchical problem, the minimization of the makespan (the time at which the vehicle returns to the depot at the end of the tour) and the minimization of the weighted sum of the times required to complete the service at each cluster. Cabral et al. also showed how the HCPP with linear precedence relations and no assumption on clusters connectivity can be transformed into an RPP and then solved by using exact and heuristic algorithms for the RPP. Perrier, Langevin, and Amaya (2008) introduced a multi-vehicle HCPP that takes into account general precedence relation constraints with no assumption on clusters connectivity, different service and deadhead speed possibilities, separate pass requirements for multi-lane road segments, class upgrading possibilities, vehicle road segment dependencies, turn restrictions, load balancing constraints, and tandem service requirements. They proposed a mathematical formulation where, in order to avoid disconnected subtours, they introduced flow variables for each arc, each vehicle and each cluster. Even for small instances of the problem, the model contains a large number of variables and constraints. They proposed two constructive methods and reported computational experiments using data from the City of Dieppe in Canada.

In this paper, we study the Hierarchical Mixed Rural Postman Problem (HMRPP), which generalizes the Hierarchical Chinese Postman Problem in two aspects. It is defined on a mixed graph with both arcs and edges, and the set of arcs and edges that requires a service does not need to be the whole set but only a subset of them. The links requiring service are divided into clusters indicating a predefined priority on the service. Following the specified hierarchical order, all the arcs and edges belonging to a cluster k must be serviced before the ones in any cluster m if $k < m$. Three different costs are associated with each required link: The cost of traversing the link if it has not been serviced yet, the cost of traversing and servicing it, and the cost of traversing the link after it has already been serviced. The presence of different costs is motivated by real applications. Consider, for example, snow removal operations, where removing snow on a street is a more time-consuming operation than traversing an already cleaned street. Uncleaned streets are hardly traversable and thus the cost of traversing them is the highest. Other costs ordering may find a practical justification as well (see Cabral et al., 2004).

As far as we know, the HMRPP has only been studied in Colombi, Corberán, Mansini, Plana, and Sanchis (2015). However, the concept of Hierarchical Rural Postman Problem is already present in Perrier, Amaya, Langevin, and Cormier (2006) and Perrier et al. (2008). In these papers, the authors developed two-phase constructive methods for the problem of designing routes for plowing operations, which, after determining a partition of the arcs to be serviced into clusters, solve a Hierarchical RPP on each cluster. In Colombi et al. (2015), we have proposed a mathematical programming formulation for the HMRPP, a matheuristic that solves sequentially a variant of the Mixed Rural Postman Problem (MRPP) for each cluster, and an effective Tabu Search algorithm that improves the result obtained by the matheuristic.

In this paper, from the formulation proposed in Colombi et al. (2015), we obtain a deep polyhedral analysis of the problem. We state conditions for the inequalities in the formulation to induce facets of the HMRPP polyhedron and study new classes of valid inequalities, such as the parity and the K-C inequalities, specifying under which conditions they are facet-inducing. Moreover, we have designed and implemented a branch-and-cut algorithm confirming that producing an effective exact method for a hard combinatorial optimization problem benefits from a good understanding of its associated polyhedron.

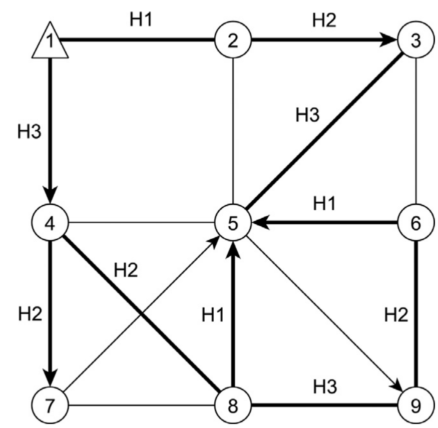

Fig. 1. An HMRPP instance.

The paper is organized as follows. In Section 2, we provide the formal definition of the problem and describe the mathematical formulation we introduced in Colombi et al. (2015). In Section 3, we analyze the relationship between the HMRPP on a graph G and the MRPP defined on an extended graph $\bar{G}$ consisting of $p+1$ copies of G, with p equal to the number of clusters. Section 4 is devoted to the study of the HMRPP polyhedron. We present different families of valid inequalities and provide conditions for them to be facet inducing. Sections 5 and 6 describe the implemented branch-and-cut algorithm and the computational results on benchmark instances, respectively. Finally, some conclusions are drawn in Section 7.

2. Problem definition and formulation

Let $G = (V, E, A)$ be a strongly connected mixed graph, where V is the set of vertices, E is the set of edges, and A the set of arcs. Vertex 1 is the depot. We will use the term *link* to refer both to an edge or an arc, indistinctly. Consider a set of required edges $E_R = E_R^1 \cup E_R^2 \cup \cdots \cup E_R^p$ and a set of required arcs $A_R = A_R^1 \cup A_R^2 \cup \cdots \cup A_R^p$ that must be serviced in a hierarchical order, that is, all the edges and arcs in $E_R^k \cup A_R^k$ must be serviced before the ones in $E_R^m \cup A_R^m$ if $k < m$.

We consider three different costs associated with each required link $(i, j) \in E_R \cup A_R$:

- $\hat{c}_{ij}$: the cost of traversing (i, j) if it has not been serviced yet,
- $\bar{c}_{ij}$: the cost of traversing and servicing it,
- $\tilde{c}_{ij}$: the cost of traversing (i, j) if it has already been serviced, and

a cost c_{ij} associated with each non-required link $(i, j) \in E_{NR} \cup A_{NR}$, representing the cost of traversing it, where $E_{NR} = E \setminus E_R$ and $A_{NR} = A \setminus A_R$. The cost of traversing an edge is assumed to be the same in both directions. We do not assume here any ordering of the costs $\hat{c}_{ij}$, $\bar{c}_{ij}$, and $\tilde{c}_{ij}$.

The HMRPP consists of finding a minimum cost closed walk (tour) starting and ending at the depot, and servicing all the required links in the hierarchical order. Fig. 1 shows an HMRPP instance with $|V| = 9$, $|E_R| = 5$ and $|A_R| = 5$, where the required links are represented in bold lines and are clustered into three clusters labeled $H1$, $H2$, and $H3$.

Note that the subgraph induced by the required links, $G(E_R \cup A_R)$, and the subgraphs $G(E_R^k \cup A_R^k)$, for all k, do not need to be connected. Note also that, since the required links must be serviced in a hierarchical order, a tour for the HMRPP can be interpreted as a series of consecutive paths, each one servicing the required links in each cluster, plus an extra path from the last vertex visited in the last cluster p to the depot.

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