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Discrete Optimization

Regenerator location problem: Polyhedral study and effective branch-and-cut algorithms

Xiangyong Li[a,*], Y. P. Aneja[b]

[a] School of Economics & Management, Tongji University, Shanghai 200092, China
[b] Odette School of Business, University of Windsor, Windsor, Ontario N9B 3P4, Canada

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ABSTRACT

In this paper, we study the regenerator location problem (RLP). This problem arises in optical networks where an optical signal can only travel a certain maximum distance (called the optical reach) before its quality deteriorates, needing regenerations by regenerators deployed at network nodes. The RLP is to determine a minimal number of network nodes for regenerator placement, such that for each node pair, there exists a path of which no sub-path without internal regenerators has a length greater than the optical reach. Starting with a set covering formulation involving an exponential number of constraints, reported and studied in Rahman (2012) and Aneja (2012), we study the facial structure of the polytope arising from this formulation, significantly extending known results. Making use of these polyhedral results, we present a new branch-and-cut (B&C) solution approach to solve the RLP to optimality. We present a series of computational experiments to evaluate two versions of the proposed B&C approach. Over 400 benchmark RLP instances, we first compare them with an available exact method for the RLP in the literature. Because of the equivalence among the RLP, the minimum connected dominating set problem (MCDSP), and the maximum leaf spanning tree problem (MLSTP), we further compare our approaches with other available exact algorithms using 47 benchmark MCDSP/MLSTP instances.

1. Introduction

In *all-optical networks*, *optical-bypass* is used to carry the traffic from a source s to a destination t entirely in the optical domain so that no Optical-Electrical-Optical (O/E/O) is required in the intermediate nodes of the path between s and t. Since the optical reach (the maximum distance an optical signal can travel before its quality deteriorates to a level that necessitates regeneration), ranges from 500 to 2000 miles (Simmons, 2006), regeneration of optical signals is essential to establish lightpaths of length greater than the optical reach. In practice, the 3R signal regeneration process is used to re-amplify, reshape and re-time the signal for wide-area backbone networks. Such regenerators are rather expensive equipment (e.g., $160K, see Mertzios, Sau, Shalom, & Zaks (2012)), and much research has been conducted, concerning minimizing their usage while satisfying all or most of the communication requirements posed by the clients. The cost of regenerators in a network is measured in two main ways: (1) the number of regenerators placed in the network, and (2) the number of locations in which regenerators are placed (Hartstein, Shalom, & Zaks, 2013). The second measure is the one that has been used in most of the research on the regenerator location problem (RLP) (Chen, Ljubić, & Raghavan, 2010; Pedrola et al., 2013; Sen, Murthy, & Bandyopadhyay, 2008; Yetginer & Karasan, 2003). The RLP deals with a constraint on the geographical extent of signal transmission in the optical network design. Fig. 1 presents an example of the RLP. In this six-node network, the optical reach is $d_{\max} = 200$ for the signal travel. For this instance, the optimal solution is to place one regenerator at node 4. With this regenerator placement, each node pair can communicate with each other.

In this paper, we focus on the RLP that can be defined as follows.

Definition 1. Given an optical network and the optical reach $d_{\max}$, the RLP is to determine the minimum number of network nodes for regenerator placement, such that for each node pair, there exists a path of which no sub-path without internal regenerators has a length greater than the optical reach $d_{\max}$.

Given an undirected graph $G = (V, E)$, a subset $D \subseteq V$ is a dominating set if every vertex $v \in V \backslash D$ is joined to at least one member of D by an edge in E. A connected dominating set is a dominating set D such that the subgraph $G(D) = (D, E(D))$ is connected,

* Corresponding author.
E-mail addresses: xyli@tongji.edu.cn (X. Li), aneja@uwindsor.ca (Y.P. Aneja).

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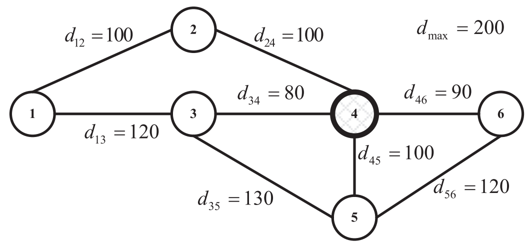

Fig. 1. Example of the RLP.

where $E(D)$ is the set of edges of E with both ends in D(Buchanan, Sung, Boginski, & Butenko, 2014). The minimum connected dominating set problem (MCDSP) consists of finding a connected dominating set of minimum cardinality. The MCDSP is $\mathcal{NP}$-hard since the problem of finding a minimal cardinality dominating set is known to be $\mathcal{NP}$-hard (Garey & Johnson, 1979). It is easy to see, as observed in Gendron, Lucena, Cunha, and Simonetti (2014) and Lucena, Maculan, and Simonetti (2010), that the MCDSP is equivalent to the maximum leaf spanning tree problem (MLSTP), which consists of finding a spanning tree of G with maximum number of leaf nodes. It was shown in Sen et al. (2008) that the RLP is equivalent to the MCDSP, and is hence $\mathcal{NP}$-hard. Hartstein et al. (2013) examined the parameterized complexity of the RLP and presented several fixed parameter tractability results and polynomial algorithms for fixed parameter values, as well as several $\mathcal{NP}$-hardness results.

Since the RLP, MCDSP and MLSTP are equivalent, solution approaches for any one of these problems can be used for the other two problems as well.

Fujie (2004) presented two binary integer programming "non-compact" formulations for the MLSTP — an "edge-vertex" formulation with $|E| + |V|$ binary variables, and a "vertex" formulation with only $|V|$ binary variables, and studied the facial structure of each of these two polytopes obtained by taking the convex hull of feasible solutions of these formulations. However, no separation algorithm or computational experiments were presented in the paper. Many practical applications of the MLSTP/MCDSP (equivalent to the RLP) are discussed in Lucena et al. (2010) and Gendron et al. (2014). Since the problem is $\mathcal{NP}$-hard, researchers have studied exact algorithms (Chen et al., 2010; Fan & Watson, 2012; Fujie, 2003; Gendron et al., 2014; Lucena et al., 2010; Simonetti, Salles da Cunha, & Lucena, 2011), heuristics (Chen et al., 2010; Duarte, Martí, Resende, & Silva, 2014; Lucena et al., 2010; Sen et al., 2008; Yue, Li, Wei, & Lin, 2014), and approximation algorithms (Flammini, Marchetti-Spaccamela, Monaco, Moscardelli, & Zaks, 2011; Guha & Khuller, 1998) for solving these problems.

Given that the investigation focus of this paper is on developing an exact algorithm, we here review works only regarding exact algorithms for the problem. For updated progress of heuristics, please refer to Duarte et al. (2014) and Gendron et al. (2014).

Fujie (2003), based on the "edge-vertex" formulation of the MLSTP discussed in Fujie (2004), presented a branch-and-bound (B&B) algorithm for the problem and reported results of some computational experiments. Lucena et al. (2010) studied two binary integer programming formulations of the MLSTP. The first formulation significantly enhanced the "edge-vertex" formulation of Fujie (2003) by adding some valid inequalities and some facet defining inequalities introduced in Fujie (2004). The second formulation recast the MLSTP as a Steiner arborescence problem on a modified directed graph, and was shown to be computationally superior to the one in Fujie (2003). Chen et al. (2010) studied the RLP, formulated it also as a Steiner arborescence problem on a modified directed graph with a unit degree constraint on the root node, and developed a branch-and-cut (B&C) algorithm. The computational results in Chen et al. (2010) show that their B&C method could optimally solve instances with up to 100 nodes and a few 200- and 300-node instances with small network density. Rahman (2012), and Rahman, Bandyopadhyay, and Aneja (2015) also studied a set covering formulation, which is equivalent to the "vertex" formulation discussed in Fujie (2004). With this formulation, the authors presented a computational study using a preliminary B&C approach (no comparison was made with any other approach).

Simonetti et al. (2011) considered the MCDSP and presented a binary integer programming formulation, using $|V| + |E|$ binary variables, that embedded two structures: one corresponding to the tree polytope for the dominating set of nodes containing the generalized subtour elimination constraints (GSEC), and the other involving the covering constraints corresponding to each node being connected to at least one of the nodes in the dominating set. The authors further strengthened their formulation by lifting both covering constraints and GSEC constraints. Another set of valid cut-inequalities were derived by observing that whenever S and $V\backslash S$ are non-dominating set of nodes, at least one edge across the cut $(S, V\backslash S)$ must be chosen in the solution tree. Fan and Watson (2012) presented compact formulations for the MCDSP and solved these formulations using CPLEX 12.1. Fernau et al. (2011) developed an exact approach, which can solve the MCDSP in $O(1.8966^n)$ time, but did not present computational results. Gendron et al. (2014) investigated further the branch-and-cut algorithm developed by Simonetti et al. (2011), and presented a Benders decomposition algorithm, a branch-and-cut method and a hybrid algorithm combining these two algorithms. The Benders decomposition approach was applied to a formulation involving only $|V|$ binary variables that contained covering constraints as described in Simonetti et al. (2011), and a generic set of constraints imposing connectedness on the selected dominating set. The branch-and-cut algorithm developed in Gendron et al. (2014) was based on strengthening further the formulation in Simonetti et al. (2011) by generalizing the cut-inequalities in Simonetti et al. (2011). Based on 47 benchmark MCDSP/MLSTP instances, Gendron et al. (2014) tested two variants of each of the above three approaches, Benders, B&C and hybrid: a stand-alone version and an iterative probing variant. The computational results showed that the methods in Gendron et al. (2014) are the current best exact approaches for the MCDSP/MLSTP and RLP.

Additionally, Guha and Khuller (1998) provided approximation algorithms for the MCDSP while Flammini et al. (2011) focused on theoretical analysis of regenerator placement, presenting exact algorithms, $\mathcal{NP}$-hardness results, approximation algorithms and hardness of approximation results for various extensions of the RLP. Mertzios et al. (2012) studied a special case of the RLP in which k possible traffic patterns are given and the objective is to place the minimum number of regenerators satisfying each of these patterns. The authors proposed a constant-factor approximation algorithm with ratio $\ln(w \cdot k)$, where w is the maximum allowed number of hops for any lightpath. Additionally, Mertzios, Shalom, Wong, and Zaks (2011) and Shalom, Voloshin, Wong, Yung, and Zaks (2012) studied the regenerator placement in an online setting.

In the computer science and network traffic literature, researchers also studied some problems related to the RLP. Yang and Ramamurthy (2005b) and Pachnicke, Paschenda, and Krummrich (2008) considered the problem of regenerating lightpaths in conjunction with the one of satisfying the greatest possible number of communication requests, given a limited number of wavelengths. Sriram, Griffith, Su, and Golmie (2004) dealt with regenerators with slight different capabilities. Yang and Ramamurthy (2005a) studied the wavelength routing under sparse regeneration in translucent optical networks. Gouveia, Patrício, De Sousa, and Valadas (2003) addressed a multi-protocol label switching (MPLS) over wave division multiplexing (WDM) network design

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