European Journal of Operational Research 257 (2017) 84–88

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

An EOQ model for perishable goods with age-dependent demand rate

Gregory Dobson[a], Edieal J. Pinker[b], Ozlem Yildiz[a,*]

[a] Simon Business School, University of Rochester, Rochester, New York, United States
[b] School of Management, Yale University, New Haven, Connecticut, United States

ARTICLE INFO

Article history:
Received 6 November 2015
Accepted 30 June 2016
Available online 15 July 2016

Keywords:
Inventory management
Perishable EOQ
Consumer behavior
Grocery industry

ABSTRACT

We study the inventory management decisions of a retailer selling a single perishable good in a deterministic setting. We take into account consumers' assessment of quality over the lifetime of the products, and assume that the demand rate is a linearly decreasing function of the age of the products. We analytically obtain the optimal cycle length of the retailer. Using our model, we obtain traditional non-perishable Economic Order Quantity (EOQ)-like lower and upper bounds on the cycle length and the profit, and show that they lead to near-optimal results for our typical examples, which are grocery items. We show that a perishable good acts similarly to a non-perishable good with unit holding cost equal to the ratio of contribution margin to lifetime. We also approximate the contribution margin the perishable good needs to have to maintain profitability parity with non-perishable goods.

1. Introduction

In this paper we formulate and analyze an EOQ model with demand decreasing in the age of the product. The model is motivated by the increased consumer interest in fresh products in the grocery sector (NPD, 2014). In fact this is not only an issue for fresh produce but for packaged goods as well. Even though the consumer may not directly observe the freshness of packaged goods, health-conscious consumers are now paying closer attention to the sell-by date (IFL, 2014). Thus, although the utility gained from consuming, for example, a can of fruit, is the same until its sell-by date, rational consumers prefer the one with more distant sell-by date because they can store it for longer. As a result, for many kinds of perishable goods, the quality assessed by the consumer gradually decreases as items age.

There are numerous models for perishable inventory that consider various structures of lifetime, demand rate, backlogging etc. See Bakker, Riezebos, and Teunter (2012) and Goyal and Giri (2001) for excellent reviews and Chern, Yang, Teng, and Papachristos (2008); Padmanabhan and Vrat (1995),Ferguson, Jayaraman, and Souza (2007),Moon, Giri, and Ko (2005), Dye and Ouyang (2005) and Chang, Teng, and Goyal (2010) for recent studies in this field. We develop a deterministic EOQ style model for perishable goods that directly incorporates consumer behavior towards freshness (see Weiss, 1982 and Pentico and Drake, 2009 for other studies using deterministic EOQ models). As the perishable good deteriorates, its quality, and thus its utility to consumers, decreases throughout its lifetime. At the end of its lifetime, the good spoils completely and has no utility for consumers. Based on this model, we derive a simple EOQ like approximation for the optimal order cycle length that is very accurate for parameter values relevant to fresh groceries. This approximation can serve as a building block for models of the fresh food supply chain and distribution systems.

To the best of our knowledge, there are only a few papers in the perishable inventory literature that take into account the decreasing utility of perishable goods throughout their lifetime. This is considered in Fujiwara and Perera (1993) for the first time. In order to include decreasing utility in their EOQ model, they introduce a penalty cost that increases over time and reaches the value of a fresh perishable product at the end of its lifetime. However, they use a demand rate that is constant over the replenishment cycle, which does not consider consumers' response to freshness. Amorim, Costa, and Almada-Lobo (2014) studies production planning models for multiple perishable products whose demands are age dependent. Their mixed-integer programming model is complex; thus, they provide only computational results. Freshness is directly captured in demand functions in Bai and Kendall (2008),Bai, Burke, and Kendall (2008) and Piramuthu and Zhou (2013) as well in a shelf space allocation context. However, due to the complexity of the problems, they focus on algorithms that give good quality solutions and provide numerical results.

* Corresponding author.
E-mail addresses: greg.dobson@simon.rochester.edu (G. Dobson), edieal.pinker@yale.edu (E.J. Pinker), ozlem.yildiz@simon.rochester.edu (O. Yildiz).

http://dx.doi.org/10.1016/j.ejor.2016.06.073
0377-2217/

Such complex objective functions make it difficult to see the structural impact of perishability. Hence, we focus our attention on a deterministic setting where demand rate linearly decreases as the products age. These simplifications allow us to reach structural results on the optimal cycle length and the profit, which are provided in Section 2. In Section 3 we analytically compare our perishable good model to the traditional EOQ model through traditional EOQ-like approximations of the perishable good cycle lengths and profits. We conclude in Section 4.

2. A model for a perishable good

We assume that D consumers come to the store every day, and are heterogeneous in their utility assessment of the product. We refer to D as the peak demand rate. We assume that each consumer has a threshold and if the perishable good is older than that threshold he will purchase nothing. For simplicity, we assume that consumers are uniformly distributed in their thresholds on $[0, \tau]$, where τ is the lifetime of the product beyond which no consumer will buy. Thus, on a day t in the cycle where t is in $[0, \tau]$, $D(1-t/\tau)$ consumers will buy and $D(t/\tau)$ will not. To simplify, we consider both time and demand to be continuous; and thus, we have the demand rate function given in (1).

$$d(t) = \begin{cases} D\left(1-\dfrac{t}{\tau}\right) & \text{if } 0 \le t \le \tau, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

This simplified form of demand function is similar to those used in Chen, Min, Teng, and Li (2016) and Wu, Chang, Teng, and Al-khateeb (2015).

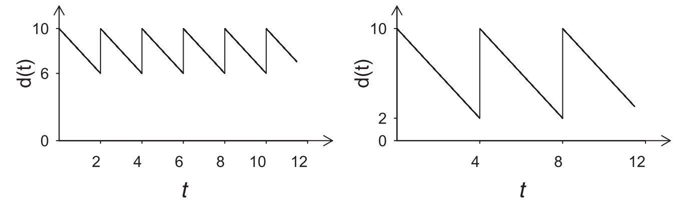
We model the retailer as maximizing the average long-run profit by deciding on the cycle length c, i.e., the time between inventory replenishments. We assume the demand rate function is known to the retailer and he incurs a holding cost of h dollars per unit time for each unit in stock. The contribution margin p is defined as the unit price less the purchasing cost of a good. Replenishments cost K dollars per cycle and are assumed to be instantaneous. We assume that these parameters are constant over time, so the ending inventory level is zero for each order cycle. As all cycles are identical, we can focus our analysis on a single cycle that begins with a replenishment at time 0.

Remark 1. Even when the retailer is not required to meet the entire demand throughout the cycle, he chooses to do so. We refer the reader to §EC1 of our e-companion paper for details. Because shortages are suboptimal in our deterministic setting, we only focus on the case in which the retailer meets the demand throughout the cycle.

We refer to goods that lead to positive optimal profits as *economically viable goods*. Throughout this paper, we focus our analysis on only economically viable goods.In addition, in §EC2 of our e-companion paper, we make clear why the retailer replenishes before the age of the product exceeds τ and always experiences a positive demand rate. Thus, all functions and the optimization problem are defined for cycle length, c, in the interval $(0, \tau]$ in the rest of this section. The cumulative quantity of products sold by time t (referred to as *the quantity sold by time t* from here on), $Q(t, c)$, and the order quantity, $Q(c, c)$, are

$$Q(t,c) = \int_0^t d(u)du = D\left(1-\frac{t}{2\tau}\right)t, \ t \in [0,c],$$
$$Q(c,c) = D\left(1-\frac{c}{2\tau}\right)c. \quad (2)$$

Because the retailer orders fresh products (with age zero) and does not have any products left at the beginning of each cycle, the cycle length c determines the age of the products on the shelf at a given time and the average quantity demanded in a cycle. For instance, with lifetime, τ, of 5 days and peak demand rate, D, of 10 per day, the retailer would observe the demand rate curves in Fig. 1a (with $c = 2$ days) and 1b (with $c = 4$ days). Inventory level at time t, $I(t, c)$, and the total holding cost in a cycle of length c, $H(c)$, are given as:



(a) $c = 2$ (b) $c = 4$

Fig. 1. Demand rates observed over the cycle.

$$I(t,c) = Q(c,c) - Q(t,c) = D(c-t) - \frac{D}{2\tau}(c^2 - t^2), \ t \in [0,c],$$
$$H(c) = \int_0^c hI(t,c)\,dt = hc^2 D\left(\frac{1}{2} - \frac{c}{3\tau}\right).$$

The retailer seeks to maximize the average long-run profit, i.e., average revenue less average holding cost and average replenishment cost. The retailer's average profit maximization problem is

$$\max_{0<c\le\tau} \Pi(c) \equiv \max_{0<c\le\tau} \frac{1}{c}(pQ(c,c) - H(c) - K)$$
$$\equiv \max_{0<c\le\tau} \frac{1}{c}\left(pDc\left(1-\frac{c}{2\tau}\right) - hDc^2\left(\frac{1}{2}-\frac{c}{3\tau}\right) - K\right). \quad (3)$$

Remark 2. The model developed above is the perishable good version of the traditional EOQ model, i.e., the EOQ model for the nonperishable items that have a constant demand rate over time (see Wilson (1934) for details). We use subscript n to refer to the nonperishable product through this paper. Recall that in the traditional EOQ model the average profit as a function of the cycle length, $\Pi_n(c)$, the optimal cycle length, c_n^*, and the optimal profit, $\Pi_n(c_n^*)$, are defined as

$$\Pi_n(c) = \frac{1}{c}\left(pDc - \frac{hDc^2}{2} - K\right),$$
$$c_n^* = \sqrt{\frac{2K}{hD}}, \ \ \Pi_n(c_n^*) = pD - \sqrt{2KhD}. \quad (4)$$

For a more detailed comparison of our perishable EOQ model and the traditional EOQ model, we refer the reader to §EC3. The retailer's average profit function $\Pi(c)$ as a function of cycle length c can have two different forms as illustrated in Figs. 2a and 2b for economically viable goods. Proposition 1 defines the optimal solution to the retailer's average profit maximization problem defined in (3).

Proposition 1. *The optimal cycle length of the retailer, c^*, is equal to the unique element of the set*

$$\left\{c \;\middle|\; -\frac{3pD + 3hD\tau - 4hDc}{6\tau} + \frac{K}{c^2} = 0 \text{ and } c \in \left(0, \min\left\{\tau, \sqrt[3]{\frac{3\tau K}{Dh}}\right\}\right]\right\}. \quad (5)$$

The proofs of our results are provided in e-companion of our paper. By Proposition 1, there exists a unique optimal cycle length in the interval $\left(0, \min\{\tau, \sqrt[3]{\frac{3\tau K}{Dh}}\}\right]$. The retailer reorders before the age of the last unit sold in a cycle reaches τ, when the demand