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Decision Support

Replacement decisions with multiple stochastic values and depreciation

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ABSTRACT

We develop an analytical real-option solution to the after-tax optimal timing boundary for a replaceable asset whose operating cost and salvage value deteriorate stochastically. We construct a general replacement model, from which seven other particular models can be derived, along with deterministic versions. We show that the presence of salvage value and tax depreciation significantly lowers the operating cost threshold that justifies (and thus hastens) replacement. Although operating cost volatility increases defer replacement, increases in the salvage value volatility hasten replacement, albeit modestly, while increases in the correlation between costs and salvage value defer replacement. Reducing the tax rate or depreciation lifetime, or allowing an investment tax credit, yield mixed results. These results are also compared with those of less complete models, and deterministic versions, showing that failure to consider several stochastic variables and taxation in the replacement process may lead to sub-optimal decisions.

1. Introduction

For assets with a significant second-hand market value, such as vehicles, earth moving equipment and aircraft, or a notable scrap value such as ships, salvage value may be a crucial ingredient to the replacement decision because of the cash flow implications. The analytical solution to the after-tax optimal timing boundary is developed for a replaceable asset characterized by a deteriorating and stochastic operating cost and salvage value. Since, at replacement, the after-tax salvage value for the incumbent plus any residual depreciation tax credits partly offset the re-investment cost, the replacement policy reflects the after-tax trade-off between the sacrificed value of the incumbent and the net benefits rendered by the succeeding asset.

From simulations on a deterministic model, Robichek and Van Horne (1967) show that abandonment can significantly raise the project value because of the flexibility value embedded in the released funds. Enhancements are made by Dyl and Long (1969) by introducing a timing option, and by Gaumitz and Emery (1980) and Howe and McCabe (1983). In a stochastic dynamic programming formulation, Bonini (1977) models a stochastic operating cost and salvage value, explicitly.

There are various empirical studies of the parameter values used for replacement models. Rust (1987) derives the drifts of operating costs (including maintenance) from actual records, which are a deterministic function of time/age. Lai, Leung, Tao, and Wang (2000) fit various lifetime distributions (including normal) to maintenance records, but do not calibrate uncertainty. Keles and Hartman (2004) quantify operating costs, salvage value and investment costs discounted to time zero for bus fleets. Kulp and Hartman (2011) study different depreciation methods, but do not consider salvage value. Liu, Wu, and Xie (2015) consider failure interaction for free-replacement warranties, van de Heijden and Iskandar (2013) evaluate spare parts storage as an alternative to replacement with higher priced versions in warranties, and Shafiee and Chukova (2013) suggest for future research modeling reliability improvements as random variables. In a review of literature, Hartman and Tan (2014) note that there only a few models which consider stochastic deterioration in continuous-time.

We observe that most real-option models which allow for stochastic variables, treat abandonment only implicitly. Salvage value and depreciation are interpreted by Mauer and Ott (1995) as functions of a stochastic operating cost as a way of reducing dimensionality to one, while Dobbs (2004) embeds the salvage value into a one-factor model. Ye (1990) allows combined maintenance and operating cost to follow an arithmetic Brownian motion,

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with a fixed investment cost, no salvage value or depreciation. Yilmaz (2001) considers revenue produced by equipment to be stochastic, but maintenance to fix any equipment faults is deterministic. Richardson, Kefford, and Hodkiewicz (2013) allow for time-to-build for a one factor replacement model. These simplifications yield an analytical solution, but any trade-offs or co-variation amongst the factors is entirely ignored.

A tractable solution for dealing with the two-stochastic-factor replacement model is developed by Adkins and Paxson (2011) that excludes depreciation and salvage value. Zambujal-Oliveira and Duque (2011) propose a two-factor model with stochastic operating cost and (autonomous) stochastic salvage value, with depreciation following a negative exponential function. Reindorp and Fu (2011) consider two stochastic factors (market price and profitability, which are uncorrelated) with investment cost a function of these two factors with no salvage (or demolition) value. Two-factor models are proposed by Adkins and Paxson (2013a,b), who consider the effect of three alternative depreciation schedules and technological progress on the replacement policy, respectively, but ignore salvage value. Adkins and Paxson (2013c) consider reversionary revenue and cost levels with technological progress, but in a deterministic framework. Ansaripoor, Oliveira, and Liret (2014) evaluate two input factors over a low-medium-high range in determining reacting to new equipment innovations. Chronopoulous and Siddiqui (2015) model different reactions to technological innovation under uncertainty, but ignore taxes and salvage values.

In this paper, we formulate a three-factor real-option replacement model, based on operating cost, salvage value and depreciation, to investigate the effect on the replacement policy not only from including salvage value as a factor but also from their interactions. The merit of our approach is the ease in determining solutions without engaging in onerous, less transparent numerical methods such as Monte-Carlo or finite-differences. Since the solution to the replacement timing-boundary is quasi-analytical, it also solves the one- and two-factor derivative models.

Both salvage value and depreciation matter in replacements, which is intuitive, but expected operating cost and salvage value volatilities and correlation also matter, not considered in deterministic models. Our general model encompasses several other models, and enables easy comparisons of the results of different models. The number of possible replacements matters a lot, extending the approximate replacement timing from 25 (multiple) to 38 (single) years for our base case parameter values given certain assumptions.

The rest of the paper is organized as follows. In Section 2, we develop a quasi-analytical method for identifying the after-tax optimal timing boundary for the three-factor replacement model. Numerical illustrations provided in Section 3 reveal significant features of the model, which are extended through a sensitivity analysis in Section 4. Section 5 concludes and offers some suggestions for further research.

2. Replacement opportunity with salvage and tax depreciation

2.1. Valuation function

We determine the real-option replacement policy for a durable productive asset, subject to input decay in a seemingly monopolistic situation whose output yields a constant revenue[1], assuming other flexibilities are inadmissible. Holding the asset remains optimal until, on an after-tax basis, the expected benefit of acquiring a successor net of replacement cost less any disposal value exceeds that from operating the incumbent. The relevant cash flows crucial to the replacement decision are those associated with the operating costs, the depreciation charge and the salvage value. While annual operating cost and salvage value, denoted by C and S, respectively, are treated as stochastic factors, the annual depreciation charge, denoted by D, is a deterministic factor. The replacement policy, represented by an optimal timing boundary separating the decision regions of continuance and replacement, is defined over a three-dimensional cost-salvage-depreciation (C-S-D) space. The tax rate τ is applicable to all cash flows (except for the investment cost), both positive and negative, and regardless of whether they represent income or capital gains. At replacement, the operating cost, salvage value and depreciation level for the newly installed succeeding asset are set to their known initial levels of C_I, S_I and D_I, respectively. The replacement re-investment cost is a known constant K. To avoid round-tripping, $S_I < K$. Asset re-investment is treated here as partly irreversible, since the firm recovers only a fraction of the original outlay if the asset is disposed at S. We assume that the revenue produced by the asset remains at a constant known level, with the restriction that it exceeds operating cost, thus insuring sufficient taxable income.

The two uncertain factors are assumed to follow distinct geometric Brownian motion processes with drift. For $X \in \{C, S\}$:

$$dX = \alpha_X X dt + \sigma_X X dz_X, \tag{1}$$

where α_X is the instantaneous drift rate, σ_X the instantaneous volatility rate, and dz_X is the increment of the standard Wiener process. Dependence between the two factors is described by the instantaneous covariance term $\rho\sigma_C\sigma_S$, $\mathrm{Cov}[dC, dS] = \rho\sigma_C\sigma_S CSdt$ with $|\rho| \le 1$. As the asset efficiency deteriorates with usage and age, we assume that the expected operating cost change α_C is positive, measured as an annualized continuous rate; correspondingly, its salvage value declines with an expected change rate of $\alpha_S \le, \ge 0$, depending on the salvage value characteristics. In contrast to previous formulations, salvage value is not directly tied to the revenue and/or operating cost of the asset, since different second-hand buyers in different countries may have little concern with the revenue or operating costs of the previous owner, and often salvage value reflects scrap value like in ships, rather than current use value.

The selected tax depreciation schedule is declining-balance, mainly because of its tractability[2]. This and alternative schedule forms are considered in a replacement setting by Adkins and Paxson (2013a). The depreciation level is described by the deterministic geometric process

$$dD = -\theta_D D dt, \tag{2}$$

where $0 < \theta_D < 1$ is a known constant proportional depreciation rate. Being time dependent, the time elapsed since the last replacement, or the age of the incumbent, can be deduced directly from the value of D. The principal difference between the evolutionary forms of C and S compared with D is the absence of the volatility term in (2). If the re-investment cost K is fully depreciable for tax purposes[3], then $D_I = \theta_D K$. The after-tax capital gain/loss on disposal $(S - \tau(S - D/\theta_D))$ is the gain/loss on S less the accumulated depreciation.

The asset value together with its embedded replacement option depends on the prevailing factor levels and is denoted by

[1] It is straightforward to recast the model in terms of net revenue instead of operating costs.

[2] The MACRS (GDS) schedule in the U.S. is a declining-balance method until it is more beneficial to switch to straight line (when the asset is older, and may be considered for replacement). It is feasible (but complicated) to model alternative tax depreciation schedules such as straight-line and sum-of-years-digits.

[3] This assumes there is no bonus or special depreciation, or investment tax credit, or requirement to estimate a residual salvage value (especially since that is stochastic), which could reduce the depreciation base.