European Journal of Operational Research 257 (2017) 251–260

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Real option valuation for reserve capacity

John Moriarty[a,*], Jan Palczewski[b]

[a] School of Mathematical Sciences, Queen Mary University of London, E1 4NS, UK
[b] School of Mathematics, University of Leeds, Leeds LS2 9JT, UK

ARTICLE INFO

Article history:
Received 15 July 2015
Accepted 2 July 2016
Available online 9 July 2016

Keywords:
Applied probability
OR in energy
Real option
Power system balancing
Capacity market

ABSTRACT

Motivated by the potential use of electricity storage to smooth fluctuations in supply and demand, we study the problem of writing American-type call options when the holder's exercise strategy is of threshold type (so that the time of exercise is known, but random). The writer must provide physical cover by buying and storing the asset *before* selling the option. We optimise the writer's strategy for a single option and for an infinite sequence of options, these two strategies being different. The latter is motivated by the lifetime valuation of an energy storage unit when used as reserve capacity in a power system. Our stochastic process is a Brownian motion representing the real-time system imbalance, and which we rescale to represent an imbalance price. The single option leads to an optimal stopping problem in which the principle of smooth fit may be violated and the stopping region may be disconnected. The lifetime analysis uses techniques and results for the single option to construct a certain fixed point characterising the value function.

1. Introduction

In an electrical power system, unexpected variations in both generation and load give rise to imbalance which is costly to correct. Various solutions to this challenge have been proposed, including dynamic consumer pricing mechanisms (Tsitsiklis & Xu, 2015). This paper is motivated by an alternative or complementary solution using electricity storage to smooth such variations. In particular we study the design of financial-type option contracts on this ancillary service.

It is common for the writer of a call option to hedge their position and in the Black–Scholes world this may be achieved by *dynamic delta hedging* (Hull, 2006), whereby more of the underlying stock is bought as the stock price rises, and vice versa. Where trading activity has an impact on the market price, the "buy high and sell low" nature of dynamic hedging may therefore exacerbate extreme fluctuations of market prices. If the buyer of a call option wishes to help stabilise or *balance* the market price, dynamic hedging therefore conflicts with this objective.

"Buying low and selling high", a reverse of delta hedging, is a fundamental investment strategy which can have the advantage of stabilising the market. Its design and optimisation has received recent attention in the context of quantitative finance under a variety of asset price models (see, for example, Zervos, Johnson, and Alazemi (2013); Zhang and Zhang (2008); Zhang (2001)). Suppose now that the underlying asset must be bought and stored *before* a call option can be written on it: in other words, that the option must be *physically covered*. This incentivises the option writer to buy and store the underlying when it is cheap, while the call option itself can deliver the asset when it is expensive. This requirement therefore leads to an alternative hedging strategy for the call option which is compatible with market balancing. Since the asset purchase can be timed flexibly, our study falls within the scope of real options analysis (Boomsma, Meade, & Fleten, 2012). This approach can be contrasted with studies which look at the impact of storage on price formation through direct trading in energy (see, for example, Gast, Le Boudec, Proutière, & Tomozei (2013)).

A certain agent is deemed to require supply of an asset when a stochastic process, which represents its price, first lies above a pre-determined threshold. At that point a second agent must provide one unit of the asset to the first agent, who pays a reward in exchange. The first agent may also pay an initial premium which is additional to the reward. Optimally timing both the purchase of the underlying asset and the writing of the option is an optimal stopping problem that the second agent solves.

This problem may be interpreted as the second agent writing a call option of American style on the underlying asset. Valuing such contracts by optimising the option holder's strategy is a classical application of optimal stopping theory (Peskir and Shiryaev, 2006, Chapter VII); we reverse this setup, fixing the holder's strategy and

* Corresponding author. Tel.: +442078822953.
E-mail addresses: j.moriarty@qmul.ac.uk (J. Moriarty), J.Palczewski@leeds.ac.uk (J. Palczewski).

http://dx.doi.org/10.1016/j.ejor.2016.07.003
0377-2217/

using optimal stopping theory to optimise the writer's actions. Our motivation comes from a problem of providing *reserve capacity* in power systems (see, for example, Just & Weber (2008)) and from assuming that the underlying asset is electricity in an *imbalance market*. In this motivating problem we model the price as a function of the instantaneous level of *imbalance* in the power system, that is supply minus load; the first agent is the *network operator* who uses the option (among other interventions) to keep the imbalance close to zero, and the second agent is the operator of an electricity storage facility such as a grid-scale battery. While the storage operator is concerned with maximising profit (the expected net present value of the cashflows described in the problem), the network operator is assumed to be concerned primarily with the physical stability of the power system. It is for this reason that the network operator's exercise strategy is assumed to be specified exogenously, rather than resulting from an economic optimisation.

Our setup leads to an optimal stopping problem in which the principle of smooth fit (see, for example, Peskir & Shiryaev (2006)) may be violated and the stopping region may be disconnected. The methodological approach we take for the single option is similar to that in Carmona and Dayanik (2008), which considers finite sequences of American type options. By adding a fixed point argument we go further, obtaining the optimal strategy and *lifetime* valuation when an arbitrarily large number of exercises is permitted. The fixed point is constructed using techniques and results for the single option case. Our stochastic process is a Brownian motion representing the real-time system imbalance, and which we rescale to represent an imbalance price. This lifetime analysis may be regarded as a single project valuation model for an electricity store (cf. Hach, Chyong, and Spinler (2016) and references therein).

The paper is organised as follows. Section 1.2 introduces the model with main findings summarised in Section 1.3. Section 2 analyses the single option, and in Section 3 we perform the lifetime analysis. The appendices, collected in an electronic companion, contain auxiliary results and detailed proofs for the single option setup.

1.1. Real Options approach

We employ the *Real Options* approach (see, e.g., Brennan and Schwartz (1985); Guthrie (2009)) in which the dynamics of the underlying stochastic process are under the physical (real) measure. The alternative of pricing under a martingale measure leads to delta hedging strategies of the type "buy high and sell low" (Hull, 2006), which exacerbate extreme market prices. In our motivating application, an electricity balancing market trades real-time adjustments to generation and load and the market price should be driven by a model of the system imbalance process $(X_t)_{t \geq 0}$. Through the prediction of load (see, for example, Hahn, Meyer-Nieberg, and Pickl (2009) and references therein), the imbalance process should have zero mean at all times and following Gast et al. (2013) we model X as a Brownian motion. In addition to being an approximation to other zero-mean diffusion processes over short time intervals, the choice of Brownian motion enables the explicit analysis which follows in this paper. We note here that the diffusion X has natural boundaries at positive and negative infinity, which plays a role in the methodology of Section 2.2.

1.2. Balancing markets

In order to move between the modelling of imbalance and the related question of price modelling we consider the UK Balancing Mechanism, which exists to equalise electricity supply and demand close to real time. In this market parties submit offers to increase generation or decrease consumption, and bids to decrease generation or increase consumption. National Grid, the network operator in the UK, seeks to correct the prevailing imbalance at least cost by taking the lowest-priced offers or accepting the highest-priced bids, subject to system constraints (see, for example, Elexon Limited (2015)). Fig. A.1 in the Appendix provides a histogram of the main system price obtained in this way, over a two and a half year period. This distribution of prices is not centred and is heavily skewed to the right. We take account of these empirical features in a straightforward manner by applying a convex transformation f to the imbalance process X, where

$$f(x) := D + de^{-bx}, \tag{1}$$

and $b,\ d > 0$. Thus our model has an 'imbalance price' process $(f(X_t))_{t \geq 0}$ which is a shifted exponential Brownian motion, and is simply a rescaling of the physical imbalance process. Here the minus sign in the exponent means that positive values of the imbalance correspond to the oversupply of the asset, and vice versa. We note that the price process $f(X_t)$ has a natural lower boundary at D, in the terminology of Borodin and Salminen (2012, Chapter 2), i.e., the price cannot reach it.

1.3. Main results

We derive the option writer's optimal policy under the above setup and the corresponding option value in both the single and lifetime problems. These two policies are different in general and the single option, in addition to being a 'basic unit', is of independent interest as it exhibits three different types of optimal stopping region. We show that the possible types are:

(a) a half-line,
(b) a bounded interval,
(c) a union of two disjoint intervals.

The smooth fit property may not hold at the (finite) boundaries of the optimal stopping region in each of the above cases. This variety of solution types, which is rather unusual in the literature on one-dimensional optimal stopping problems, can be anticipated: lower price levels mean a lower cost for purchasing the asset but also a longer time until the reward is received. The parameter-dependent interplay between these opposing considerations therefore determines the precise form of the solution.

In our motivating problem, energy is stored in a single battery of unit capacity and so in order to ensure delivery of the energy when needed, a second option may be written only after the first option has been exercised and the battery has been replenished. The investment value of the battery when used as reserve capacity is therefore equal to the value of an infinite sequence of such real options, which we call the 'lifetime valuation' (a somewhat related study may be found in Carmona and Ludkovski (2010), where the value of gas storage units used for price arbitrage is derived using a numerical approach). We show that the lifetime value is finite when the sum of the single contract payments (initial premium and reward) is strictly less than the imbalance price upon exercise. We obtain the writer's optimal policy and the corresponding lifetime value.

2. Single call option

In this section, we formulate the writer's optimal strategy as an optimal stopping problem (Section 2.1), present the solution method and geometric analysis of the obstacle (Sections 2.2 and 2.3) and give an overview of the set of optimal strategies depending on the parameter values, from which the option value follows directly. The detailed case by case analysis is given in the appendices (Sections C.1–C.3). We conclude by discussing whether the contract, once optimised from its writer's point of view, indeed stabilises the imbalance process $(X_t)_{t \geq 0}$, or equivalently the price process $(f(X_t))_{t \geq 0}$.