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Discrete Optimization

Two decomposition algorithms for solving a minimum weight maximum clique model for the air conflict resolution problem

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ABSTRACT

In this paper, we tackle the conflict resolution problem using a new variant of the minimum-weight maximum-clique model. The problem involves identifying maneuvers that maintain the required separation distance between all pairs of a set of aircraft while minimizing fuel costs. We design a graph in which the vertices correspond to a finite set of maneuvers and the edges connect conflict-free maneuvers. A maximum clique of minimal weight yields a conflict-free situation that involves all the aircraft and minimizes the costs induced. The model uses a different cost structure compared to classical clique search problems: the costs of the vertices cannot be determined *a priori*, since they depend on the vertices in the clique. We formulate the problem as a mixed integer linear program. Since the modeling of the aircraft dynamics and the computation of trajectories is separated from the solution process, our mathematical framework is valid for any hypotheses on the aircraft dynamics and any choice of the available maneuvers. In particular, the aircraft can perform dynamic velocity, heading, and flight-level changes. To solve instances involving a large number of aircraft spread over several flight levels, we introduce two decomposition algorithms. The first is a sequential mixed integer linear programming procedure that iteratively refines the discretization of the maneuvers to yield a trade-off between computational time and cost. The second is a large neighborhood search heuristic that uses the first procedure as a subroutine. The best solutions for the available set of maneuvers are obtained in less than ten seconds for instances with up to 250 aircraft randomly allocated to bisten flight levels.

1. Introduction

1.1. Context: challenges of air traffic control

In recent years air traffic management (ATM) has attracted increasing attention, and research has focused on advanced decision algorithms. Such automated tools will be key components of future ATM systems such as the Single European Sky ATM Research (SESAR) SESAR Joint Undertaking (2012) project in Europe and the Next Gen Joint Planning and Development Office (2008) program in the United States. Optimization algorithms for air traffic control (ATC) are particularly relevant in the current context of growing traffic, where airspace capacity and safety become concerns. The latest long-term forecast from EUROCONTROL predicts that traffic demand will increase by 20 percent to 80 percent between 2012 and 2035 (see EUROCONTROL (2013)). A simulation-based study performed by Lehouillier, Omer, Soumis, and Allignol (2014) shows that for a 50 percent increase in traffic, the controllers in charge of busy sectors would have to resolve an average of 27 conflicts per hour. Decision tools are essential in such an environment.

1.2. Literature review

A fundamental challenge of ATC is the air conflict resolution (CR) problem. A conflict occurs when two aircraft fail to respect predefined horizontal and vertical separation distances of respectively 5 nautical miles (NM) and 1000 ft, as illustrated in Fig. 1. To resolve conflicts, the controllers impose speed, heading, or altitude-change maneuvers. Given the current position, speed, acceleration, and predicted trajectory of a set of aircraft, the CR problem consists in identifying the conflict-free maneuvers that minimize a given cost function.

The CR problem has been widely studied. We provide a synthesis of the studies that had the greatest influence on our work; a

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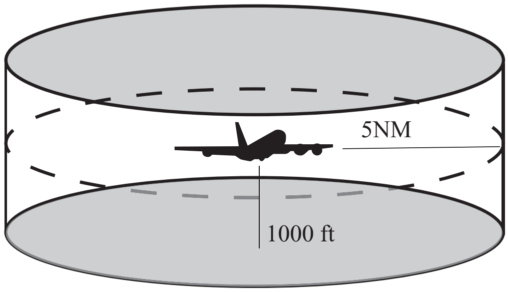

Fig. 1. Safety cylinder around an aircraft.

more complete literature review may be found in Martín-Campo's thesis (Martín-Campo, 2010). Because aircraft trajectories are time-continuous, the most natural approach is to model the problem using optimal control (Zhou, Doyle, Glover et al., 1996). Analytical solutions can be found for only the simplest cases, but the models can be solved numerically using nonlinear programming techniques. For instance, Raghunathan, Gopal, Subramanian, Biegler, and Samad (2004) use a time discretization of the problem to derive solutions for instances with more than two aircraft. One difficulty is that the nonlinear program (NLP) is nonconvex, so the global optimum cannot be found in a reasonable time and the solution is sensitive to the starting point.

Several heuristics have been developed to find feasible solutions quickly. Durand, Alliot, and Noailles (1996) and Meng and Qi (2012) develop ant colony algorithms, where maneuvers are chosen from a finite discrete set of heading changes performed at constant speed. Alonso-Ayuso, Escudero, Martín-Campo, and Mladenović (2014) adapt a variable neighborhood search algorithm and consider only heading changes. Other methods use maneuvers extracted from a prescribed set Vivona, Karr, and Roscoe (2006), particle swarm optimization (see Gao, Zhang, and Guan (2012) for heading changes), or neural networks (see Durand, Alliot, and Médioni (2000) and Christodoulou and Kodaxakis (2006) for speed changes). These methods are fast, but convergence is not guaranteed.

Mixed integer linear and nonlinear programming (Jünger et al. (2010); Wolsey (2008) and Lee and Leyffer (2011)) provide powerful theoretical frameworks for CR. With the realistic restriction that the aircraft perform at most one maneuver at the initial time, Pallottino, Feron, and Bicchi (2002) exploit the geometry of the separation constraints to develop two mixed integer linear programs (MILPs) that allow either a speed change with a constant heading or a heading change with a constant speed. Vela et al. (2011) develop an MILP that allows both speed and heading changes, and Christodoulou and Costoulakis (May 12–15, 2004, Dubrovnik, Croatia) describe a nonlinear model for three-dimensional CR. The MILP of Alonso-Ayuso, Escudero, and Martín-Campo (2011) allows both velocity and altitude changes. In Alonso-Ayuso, Escudero, and Martín-Campo (2012), Alonso-Ayuso et al. extend the model of Pallottino et al. (2002) by replacing the instantaneous speed changes with continuous changes. Schouwenaars (2006) and Omer and Farges (2013) use a time-based discretization of the optimal control formulation. Vela, Solak, Singhose, and Clarke (16–18 December 2009) and Omer (2015) develop MILPs with a space discretization that focus on the main points of interest of the CR.

In the ATM field, graph theory has primarily been used for air traffic flow management (ATFM) Bertsimas and Patterson (1998); 2000). In ATC, conflicts between aircraft are generally modeled by a graph in which the vertices represent the different aircraft and the edges link pairs of conflicting aircraft. Vela (2011) and Sherali, Cole Smith, and Trani (2002) use conflict graphs in their models. Resmerita, Heymann, and Meyer (December 2003) study *a priori* CR by developing a multi-agent system where each aircraft must choose a path in a resource graph in which the vertices represent zones of the airspace and where the chosen paths must be conflict-free. Barnier and Brisset (2004) assign different flight levels to aircraft with intersecting routes by looking for maximum cliques in a graph defining an assignment of all the aircraft to a set of given flight levels.

1.3. Contribution statement

We present a formulation of the CR problem as a variant of the minimum-weight maximum-cardinality clique (MWMCC) problem. A preliminary study is presented in Lehouillier, Omer, Soumis, and Desaulniers (2015a, 2015b). We design a graph in which the vertices represent possible aircraft maneuvers and the edges link conflict-free maneuvers of different aircraft. The innovation of this model is its cost structure. The costs of the vertices are not known *a priori* since they depend on which maneuvers are in the clique. Moreover, this approach is flexible, because the nature of the model does not depend on the modeling of the aircraft dynamics and on the available maneuvers. As a consequence, our mathematical framework remains valid for any hypotheses on the aircraft dynamics and maneuvers, on the computation of the separation distances, and on the cost evaluation. In contrast, the models that include constraints representing aircraft' dynamics are usually valid only with one set of hypotheses on dynamics and maneuvers (see Christodoulou and Kodaxakis (2006); Durand et al. (2000, 1996); EUROCONTROL (2013); Gao et al. (2012); Joint Planning and Development Office (2008); Jünger et al. (2010); Karp (1972); Lee and Leyffer (2011); Lehouillier et al. (2014,2015a,b); Omer (2015)).

We have made several significant improvements to the model in Lehouillier et al. (2015a,b). First, we have corrected the cost computation. Second, our key contribution is that we have developed two decomposition algorithms to address the explosion of the number of vertices that occurs in large instances. The first algorithm is a sequential mixed integer linear programming (SMILP) procedure that iteratively refines the discretization of the set of maneuvers without changing the number of vertices in the graph. This yields a trade-off between computational time and the cost of the optimal solution. This procedure is then used as a subroutine in a spatial decomposition that takes advantage of the geometric structure of the instances. The spatial decomposition is a large neighborhood search metaheuristic that exploits the weak interdependency between subsets of aircraft. Finally, we have tested our model on an extended benchmark that includes the structured instances with up to 20 aircraft described in Lehouillier et al. (2015a,b), and random instances with up to 60 aircraft on a single flight level and 250 aircraft over several flight levels. The results show that automated CR can be performed in a few seconds for large and dense areas of the airspace.

2. Problem formulation

In this section, we discuss the modeling of the aircraft dynamics and maneuvers, the computation of the separation distances, and the cost evaluation. The choices made in this section represent a possible modeling of the problem. However, they are independent of the solution method, so considering other possibilities would not impact the validity of our overall method.

2.1. Modeling of aircraft dynamics

As is standard in the literature, we use a three-dimensional point-mass model for the aircraft dynamics:

$$\frac{dp_x}{dt} = V \cos\gamma \cos\chi \quad (1)$$