European Journal of Operational Research 000 (2016) 1–20

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Capital- and resource-constrained project scheduling with net present value optimization

Pieter Leyman[a], Mario Vanhoucke[a,b,c,*]

[a] Faculty of Economics and Business Administration, Ghent University, Tweekerkenstraat 2, 9000 Ghent, Belgium
[b] Technology and Operations Management Area, Vlerick Business School, Reep 1, 9000 Ghent, Belgium
[c] UCL School of Management, University College London, Gower Street, London WC1E 6BT, United Kingdom

ARTICLE INFO

Article history:
Received 22 July 2015
Accepted 12 July 2016
Available online xxx

Keywords:
Project scheduling
Net present value
Capital
Metaheuristics

ABSTRACT

In this paper, we study the capital-constrained project scheduling problem with discounted cash flows (CCPSPDC) and the capital- and resource-constrained project scheduling problem with discounted cash flows (CRCPSPDC). The objective of both problems is to maximize the project net present value (NPV), based on three cash flow models. Both problems include capital constraints, which force the project to always have a positive cash balance. Hence, it is crucial to schedule activities in such an order that sufficient capital is available.

The contribution of this paper is threefold. First, we propose three distinct cash flow models, which affect the capital availability during the project. Second, we introduce two new schedulers to improve capital feasibility, one for the CCPSPDC and one for the CRCPSPDC. The schedulers focus on delaying sets of activities, which cause cash outflows to be received at later time instances, in order to reduce capital shortages. Both schedulers are implemented as part of three metaheuristics from literature, in order to compare the metaheuristics' performance. Two penalty functions have been included, one to improve capital feasibility and another to improve deadline feasibility. Third, the proposed procedure has been tested on a large dataset and the added value of the schedulers has been validated. Managerial insights are provided with respect to the impact of key parameters.

1. Introduction

Project scheduling, and in particular the resource-constrained project scheduling problem with duration minimization (RCPSP), has been the topic of a large amount of research over the past few decades. A great deal of focus has been on regular measures of performance, which are non decreasing functions of the activity finish times, as is the case for the RCPSP. Non-regular objectives, i.e. an objective for which the definition given for regular objectives does not hold, have become increasingly popular and varied. We focus on the maximization of the project net present value (NPV), which implies that a negative or positive cash flow is assigned to each activity and the project objective is to schedule all activities in such a way that the NPV is maximized. The distinction can furthermore be made between the max-NPV problem (no renewable resources are included) and the RCPSP with discounted cash flows (RCPSPDC, renewable resources are included).

In this paper, we extend the resource-unconstrained max-NPV problem with capital constraints and three cash flow models, and refer to this problem as the capital-constrained project scheduling problem with discounted cash flows (CCPSPDC). Furthermore, the problem is also extended with the presence of renewable resource constraints, and we refer to this problem as the capital- and resource-constrained project scheduling problem with discounted cash flows (CRCPSPDC). In the max-NPV problem and in the RCPSPDC no limit is set on the cash balance (the sum of the cash inflows received and the cash outflows paid) at any particular time, which implies that the cash balance may very well be negative at certain times during the project duration. The CCPSPDC and CRCPSPDC impose the additional constraints that at no point in time the cash balance, or available capital, can be negative. This way, cash outflows can only be paid if sufficient capital is available, whereas cash inflows add to this capital. We present metaheuristic solution procedures with new scheduling techniques tailored to the needs of the problems, in particular to the capital constraints. The solution procedures in this paper focus on solving capital shortages

---

* Corresponding author at: Faculty of Economics and Business Administration, Ghent University, Tweekerkenstraat 2, 9000 Ghent, Belgium.

E-mail addresses: pieter.leyman@ugent.be (P. Leyman), mario.vanhoucke@ugent.be (M. Vanhoucke).

http://dx.doi.org/10.1016/j.ejor.2016.07.019
0377-2217/

Please cite this article as: P. Leyman, M. Vanhoucke, Capital- and resource-constrained project scheduling with net present value optimization, European Journal of Operational Research (2016), http://dx.doi.org/10.1016/j.ejor.2016.07.019

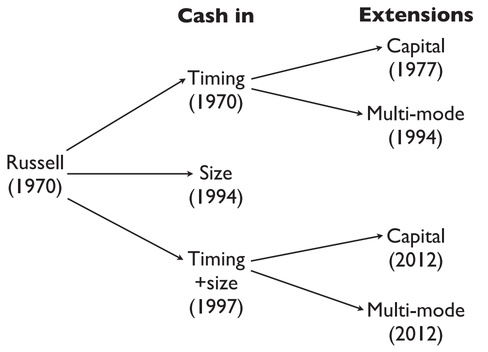

Fig. 1. History of the max-NPV problem and its extensions.

by delaying negative cash flows in order to move these cash flows later than the capital shortage.

The remainder of this paper is organized as follows. A literature overview is given in Section 2. The mathematical problem definitions of the CCPSPDC and the CRCPSPDC are discussed in Section 3, whereas two schedulers are the focus of Section 4. In Section 5 an overview of the metaheuristics employed is given, whereas the computational experiments and their results are analyzed in Section 6. We finish with a conclusion and recommendations for future research in Section 7.

2. Literature overview

Fig. 1 provides an overview of the history of the max-NPV problem, since NPV optimization in project scheduling was first introduced by Russell (1970). The years in the figure indicate when the first research on the problem (extension) was conducted. The distinction is made between three large areas of research with respect to the cash inflows of activities, which depend on the negotiations between the contractor and the client:

- **Timing:** the size of the cash inflows can be determined in advance, but the occurrence of payments depends on the actual project schedule. This is typically the case if cash inflows occur at each activity's completion time. As a result, the contractor can only control the timing of cash inflows, but the size is determined by the client.
- **Size:** the payment times are selected in advance (e.g. progress payments every 10 time units), but the size of the payments can only be determined based on the schedule. The contractor can influence the size of the cash inflows, but not their occurrence times.
- **Both:** the timing and size both depend on the project schedule. In this case, the contractor can determine both, but can only employ a limited number of payments (e.g. Dayanand & Padman, 1997), or the payment times depend on the progress of the project (e.g. based on total costs incurred by the contractor (He, Liu, & Xu, 2009a)).

Additionally, two of these research classes, namely the research on timing of cash inflows and on the combination of timing and size, have been extended to include capital and multiple activity modes. In the latter case, each activity can be executed with different time-resource combinations.

For an overview of the project scheduling literature with NPV optimization up to 1997, we refer to Herroelen, Van Dommelen, and Demeulemeester (1997). A summary of more recent work from 1997 on the max-NPV problem and its extensions is displayed in Table 1. In the *"Cash in"* columns of the table, we highlight whether the objective includes the determination of the timing and/or size of the cash inflows. The *"Extensions"* columns distinguish the papers in the table based on the inclusion of more specific problem characteristics. A trade-off between multiple modes of an activity (MM) is specified as well. At the bottom of the table, all papers that include capital constraints (C), even those published before 1997, are included since these papers solve a problem closely related to the one of this manuscript.

From the table and Fig. 1 we conclude that the majority of the research on the max-NPV problem has been on the timing of cash inflows and several extensions. Recent papers on the single- and multi-mode RCPSPDC (Leyman & Vanhoucke, 2015, 2016), however, show that no research has been done on capital restrictions in combination with renewable resource limitations.

In this paper, we contribute to the literature on NPV maximization in project scheduling in three ways. First, we propose a new scheduling technique as part of a metaheuristic approach for the max-NPV problem with capital constraints, and compare with existing work. Second, we focus on the problem with renewable resource and capital constraints, which has not yet been discussed in literature. We introduce a scheduler which handles both restrictions, while optimizing the project NPV. Third, whereas the focus in literature has been on the timing and/or size of cash *inflows*, we model the timing and size of cash *outflows* as part of a general model. These cash outflows are particularly relevant with capital constraints, since their timing and size have a profound impact on the capital level available during the project (Section 3).

3. Problem definition

In this section, we discuss the problem definitions of the capital-constrained project scheduling problem with discounted cash flows (CCPSPDC), and of its renewable resource-constrained extension the CRCPSPDC.

3.1. The capital-constrained project scheduling problem with discounted cash flows

A project can typically be represented by a directed graph or network $G(N, A)$ with N used for the project activities or nodes and A the precedence relations or arcs between the nodes N. We employ the activity-on-the-node (AoN) representation and assume a time-lag of zero for the precedence relations. Each activity i ($i \in N = \{1, \ldots n\}$) has a duration d_i, a cash inflow $c_{i,\, in}$ (>0) and a cash outflow $c_{i,\, out}$ (<0). Additionally, a start dummy 0 and end dummy $n+1$ are included. The project has a deadline of δ_{n+1}. The finish times f_i of the activities are the decision variables.

Mathematically, the max-NPV problem can be conceptually formulated as follows:

$$\text{Maximize} \quad \sum_{i=1}^{n} (c_{i,in} + c_{i,out}) \cdot e^{-\alpha f_i} \tag{1}$$

Subject to:

$$f_i \le f_j - d_j, \quad \forall (i, j) \in A, \tag{2}$$

$$f_{n+1} \le \delta_{n+1}, \tag{3}$$

$$f_i \in int^+, \quad \forall i \in N \tag{4}$$

The objective function (1) optimizes the project NPV based on a discount rate α, and discounts both the cash in- and outflow to the activity finish time. Constraints (2) enforce precedence feasibility. Constraint (3) makes sure the project deadline is met. If no deadline were imposed, cash outflows could be delayed indefinitely. Finally, constraints (4) state that the decision variables should be integer values.

In objective function (1), it is assumed that cash in- and outflows both occur at activity completion time. However, for the extension with capital constraints (CCPSPDC) we consider alternative