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Discrete Optimization

Stochastic survivable network design problems: Theory and practice

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ABSTRACT

We study survivable network design problems with edge-connectivity requirements under a two-stage stochastic model with recourse and finitely many scenarios. For the formulation in the natural space of edge variables we show that facet defining inequalities of the underlying polytope can be derived from the deterministic counterparts. Moreover, by using graph orientation properties we introduce stronger cut-based formulations. For solving the proposed mixed integer programing models, we suggest a two-stage branch&cut algorithm based on a decomposed model. In order to accelerate the computations, we suggest a new technique for strengthening the decomposed L-shaped optimality cuts which is computationally fast and easy to implement. A computational study shows the benefit of the decomposition and the cut strengthening – which significantly reduces the number of master iterations and the computational running time. Moreover, we evaluate the stability of the scenario generation method and analyze the value of the stochastic solution.

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1. Introduction

Motivation. Survivable network design problems with edge-connectivity requirements (SNDPs) are among the most fundamental problems in the field of network optimization. Many classical network design problems including the shortest path problem, the minimum spanning tree problem, the Steiner tree problem, the minimum-weight edge-connected subgraph problem, and edge-connectivity augmentation problems are special cases of the survivable network design problem. Applications of the (special cases of the) SNDP can be found in many different fields, e.g., in the design of supply chain and distribution networks or in the chip layout design (Botton, Fortz, Gouveia, & Poss, 2013; Kandyba, 2011; Kerivin & Mahjoub, 2005; Rodríguez-Martín, Salazar-González, & Yaman, 2016). The field of telecommunications belongs to the most important applications that request building cost-effective networks with higher connectivity requirements.

In a typical telecommunication network (Fortz, Mahjoub, McCormick, and Pesneau, 2006; Grötschel, Monma, and Stoer, 1992), an edge weighted graph is given with three *types of nodes*: "special" offices (which are nodes of type 2 that correspond to important hubs, business customers, or private households with high-demanding service packages), "ordinary" offices (nodes of type 1 corresponding to regular customers, like single households), and "optional" offices (nodes of type 0 representing street junctions). The goal is to find a cost-minimal subnetwork which ensures that all special offices are connected by two paths, all ordinary offices are at least simply connected, and optional offices can be used to establish connections, if that would lead to a cheaper solution. This problem is known as the {0, 1, 2}-SNDP. Here, we consider the general SNDP with arbitrary connectivity requirements between each pair of nodes.

In practice, however, from the moment that the information concerning the type of node is gathered until the moment in which the solution has to be implemented, some of the data might change with respect to the initial setting. In the present paper we define a mathematical model that helps decision makers to deal with the following two types of uncertainty:

- Uncertainty with respect to node types: Node types may change over time, subject to many external conditions. For example, socio-economic factors like customer's purchase power, recession, or inflation may influence expected customer's demand. Furthermore, changes in urban city planning or political factors can lead to changes in the demand of a whole neighborhood, or availability of a given location to host an office. Finally, multiple service providers compete for the customers such that customer demand highly depends on the competing service offers available at the market.

For all these reasons, during the strategic planning of a telecommunication network, it remains unclear which potential

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customers may be willing to subscribe to the service and to which particular service package.
- Uncertainty with respect to investment costs: The costs of establishing links (installing new cables, pipes, etc.) may be subject to inflations and price deviations. Price deviations are common due to the frequent changes in the underlying technology and (un)availability of the corresponding equipment.

Hence, a solution obtained using a classical deterministic model might become suboptimal or even infeasible once the network is deployed in which case a new solution might have to be redefined from scratch.

Despite the great importance of the SNDP and the relevance of the uncertainty for practical applications, to our knowledge, no publications are available that investigate the SNDP under these two particular sources of data uncertainty. In this paper we attempt to close this gap by considering *two-stage stochastic* versions of survivable network design problems with edge-connectivity requirements (for an introduction to stochastic programing see, e.g., Birge & Louveaux, 2011). Thereby, the uncertain data is modeled using random variables with a set of scenarios defining their possible outcomes. Typically, a solution is comprised by first- and second-stage decisions such that a partial subnetwork is built in the first stage which is then completed once the uncertain data becomes available in the second stage.

More precisely, in *the two-stage stochastic survivable network design problem* (SSNDP), network planners want to establish profitable connections now (in the *first stage*) while taking all possible outcomes – the *scenarios* – into account. In the future (in the *second stage*) the actual scenario with its requirements and connection costs is revealed and additional connections can be purchased (through so-called *recourse* actions) to satisfy the now known requirements. The objective is to minimize the *expected costs* of the solution, i.e., the sum of the first-stage costs plus the expected costs of the second stage. Thereby, all connectivity requirements for all scenarios have to be satisfied. The formal definition of the SSNDP is given in Section 3.

Previous work. There exists a large body of work on different variants of the deterministic survivable network design problem. We refer to Kandyba (2011), Kerivin and Mahjoub (2005) for a comprehensive literature overview on the SNDP. Many polyhedral studies were done in the 90's, see, e.g., Grötschel, Monma, and Stoer (1995a), Kerivin and Mahjoub (2005). A decade later the question of deriving stronger mixed integer programing (MIP) formulations by orienting the k-connected subgraphs has been considered by e.g. Balakrishnan, Magnanti, and Mirchandani (2004), Magnanti and Raghavan (2005). Among the approximation algorithms for the SNDP, we point to the work of Jain (Jain, 2001) whose approximation factor of two remains the best one up to date.

Regarding the stochastic variants there are significantly less results published so far. To the best of our knowledge, the only results about the SSNDP with general edge/node-connectivity requirements are contained in our short paper (Ljubić, Mutzel, and Zey, 2013); cf. the next paragraph. One of the investigated special cases of the SSNDP is the *two-stage stochastic Steiner tree problem* in which node types are either zero or one. For this problem approximation algorithms (see, e.g., Gupta, Ravi, & Sinha, 2007; Shmoys & Swamy, 2006), MIP approaches (see Bomze et al., 2010), and heuristics (see Hokama, Felice, Bracht, & Usberti, 2014) were developed. For the SSNDP involving node types ≥ 2, up to our knowledge, there only exists an $O(1)$ approximation algorithm (see Gupta, Krishnaswamy, & Ravi, 2009) for the following special case of the $\{0, k\}$-SSNDP: For each pair of distinct nodes i and j a single scenario, whose probability is p_{ij}, is given in which nodes i and j need to be k-edge-connected. But in general, however, it follows by Ravi and Sinha (2006) that the SSNDP is as hard to approximate as label cover – which is $\Omega(\log^{2-\epsilon} n)$ hard. In fact, the hardness-proof already works for the stochastic shortest path problem.

Besides the design of survivable networks a lot of research has been done concerning the design of reliable networks (e.g., recent papers can be found in the special issue Rak & Sterbenz, 2015). Design of reliable networks under network uncertainty using the approach of *chance-constrained programing* (see, e.g., Prékopa, 2003), has been considered in Song and Luedtke (2013), Song and Zhang (2015). In chance-constrained programing, there is usually one decision horizon (i.e., no recourse) and a feasible solution has to satisfy the constraints with a given probability. In Song and Luedtke (2013), Song and Zhang (2015) $s-t$-paths and the Steiner tree problem, respectively, have been considered under possible network failure scenarios. In contrast to the SSNDP studied in this paper, these problems assume the set of customers remains the same across all scenarios but a whole subnetwork can be subject to failure. Each failure scenario happens with a certain probability and the goal is to find a reliable network that ensures given connectivity requirements with a certain probability. The authors introduce several (M)IP formulations, facet-defining inequalities, and provide computational studies.

Our contribution. Our contribution is twofold, it concerns theoretical models as well as practical algorithms.

Theory: In the past, the seminal result of Nash-Williams (1960) has been used to develop stronger MIP models for the deterministic SNDP by exploiting graph orientations (Chimani, Kandyba, Ljubić, and Mutzel, 2010; Chopra, 1994; Magnanti and Raghavan, 2005). Here, we discuss that graph orientation properties cannot be used in a straight-forward fashion to develop similar models for the SSNDP. As an alternative, we propose two general ways to develop *semi-directed* MIP models in which only the second-stage solutions are oriented. We develop two novel cut-based MIP models of the deterministic equivalent for solving the SSNDP on undirected graphs based on these orientation properties. We prove that the new models are stronger than the original one based on standard undirected cuts. Moreover, when considering the undirected formulation of the SSNDP we show that facet defining inequalities can be easily derived from their deterministic counterparts.

Computational study: The SSNDP belongs to a broader class of two-stage integer stochastic programs with binary first-stage solutions and binary recourse. These NP-hard problems are known to be notoriously difficult to solve (Schultz, 2003). In this paper, we use a recently introduced decomposition approach called *two-stage branch&cut* (Bomze et al., 2010). This approach uses a Benders decomposition and two nested branch&cut algorithms and is similar to the integer L-shaped method (Laporte and Louveaux, 1993). In the *subproblems*, violated directed cuts are separated, while the *master problem* is expanded by L-shaped and integer optimality cuts. To enhance the algorithmic performance, we propose a new computationally inexpensive procedure that strengthens the inserted L-shaped optimality cuts by simple modifications of the dual solutions of the subproblems. To illustrate the effectiveness of the strengthening procedure, we compare our approach with the classical method by Magnanti and Wong (1981) for generating Pareto-optimal L-shaped cuts. Using a large set of realistic instances, we analyze in detail the characteristics of the proposed models and the obtained solutions as well as the performance (e.g., on denser graphs), behavior, and limitations of the designed algorithmic approach. The computational study is completed by an evaluation of the value of the stochastic solution and an analysis of the stability of the scenario generation method.

A small portion of results presented in this paper appeared in the conference proceedings (Ljubić et al., 2013). This paper offers

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