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Analytical properties of an imperfect repair model and application in preventive maintenance scheduling

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A B S T R A C T

The paper considers repairable systems under imperfect repair. The failure rate of a new system is assumed to follow a Weibull distribution and the repair efficiency is characterized by a Kijima type II virtual age model named Arithmetic Reduction of Age with infinite memory. An analytical approach to obtain the distribution of the inter-failure times is presented. The existence of a stationary regime is highlighted and the limiting distributions are explicitly derived. In this context, an optimal age-based preventive maintenance policy can be implemented. Three approaches are proposed, considering a static, a dynamic or a failure limit policy. Numerical simulations are presented to illustrate the policies.

1. Introduction

During their operational life, industrial systems are subject to repair actions when a failure occurs. A repair activity is aimed to reduce the failure rate of the system and to extend its useful lifetime. The maintenance process has to take into account both the intrinsic aging of the system and the repair effectiveness. These two elements allow a better understanding of the system behavior in the short and long terms and the maintenance policy can be adapted consequently.

Repair efficiencies are commonly assumed to be either minimal or perfect. A minimal or As Bad As Old (ABAO) repair assumes that the system is restored to its operational condition just before the failure. A perfect or As Good As New (AGAN) repair consists in restoring the system to a new and identical one. Minimal repair and perfect repair can be characterized by a non-homogeneous Poisson process and a renewal p rocess (Ascher & Feingold, 1984), respectively. However, for a repairable system, these assumptions are not always realistic as the system can be effectively repaired but is not renewed. This situation is described as imperfect maintenance (Pham & Wang, 1996). A thorough account of imperfect maintenance modeling for repairable systems is developed by Lindqvist (2006). In the context of imperfect repair, the implementation of optimal maintenance policies have been developed by Nakagawa (2005) and by Pham and Wang (2006). The optimization of imperfect maintenance policies are discussed considering reliability block diagrams (Levitin & Lisnianski, 2000) and examples of application to failure data are presented by Baker (2001) and Dijoux and Gaudoin (2014).

Virtual age models (Kijima, Morimura, & Suzuki, 1988) are the most frequently used imperfect repair models. The principle is that the wear-out does not depend on the chronological age of the system, but on a virtual age, commonly between zero and the elapsed time since the system was new. A virtual age model is entirely characterized by the failure rate of a new system and by the virtual age assumptions. In particular, Kijima (1989) has proposed two widespread classes of virtual age assumptions. He supposes that each repair efficiency is represented by a random variable supported on the interval [0,1]. A model under Kijima Type I assumption is such that a repair rejuvenates the virtual age of a proportional amount of the last inter-failure duration, whereas a model under Kijima Type II assumption supposes that the rejuvenated amount is proportional to the virtual age just before the repair. A particular case is to consider that the repair efficiency is a constant $\rho \in [0, 1]$, called restoration factor. The resulting models have been developed by Malik (1979) and by Brown, Mahoney, and Sivazlian (1983) for the Kijima type I and II models, respectively. A unified version of the last two models has been presented by Doyen and Gaudoin (2004), called model of arithmetic reduction of age with memory m, and denoted ARA_m. The ARA_1 and ARA_∞ models are special cases of the Kijima Type I and II models, respectively. Theoretical results on the ARA_1 model are developed in the literature (Kijima & Sumita, 1986; Malik, 1979; Yevkin, 2012) and are applied

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in maintenance scheduling (Dimitrakos & Kyriakidis, 2007; Jiang, Makis, & Jardine, 2001; Kijima et al., 1988; Love, Zhang, Zitron, & Guo, 2000; Makis & Jardine, 1993). Similarly, some properties of the ARA_∞ model are discussed when it is introduced (Doyen & Gaudoin, 2004; Kijima, 1989). These models are also developed in the presence of different kinds of maintenance actions (Dijoux & Idée, 2013; Doyen & Gaudoin, 2011).

Last and Szekli (1998) have proven the convergence of the Kijima Type II model, and hence of the ARA_∞ model, to a steady-state regime. Finkelstein (2008) has proven the convergence of the ARA_∞ model to a steady-state regime when the repair efficiency depends on the chronological age of the system. In contrast, Doyen (2010) has proven that the ARA_1 model behaves asymptotically as a non-homogeneous Poisson process. These properties highlight a major difference between the ARA_1 and ARA_∞ models when the restoration factor is in the interval]0, 1[. If the failure rate of a new system is increasing monotonically to infinity, the inter-failure times converge to zero for the ARA_1 model and to a stationary distribution for the ARA_∞ model.

This paper aims to solve a maintenance problem inspired by a failure data set of electrical transformers given by French electrical company (Électricité de France-EDF). Data consist of maintenance dates without any information on type of maintenance and the age of the system. The data concern systems running for a long and unknown period of time. However, the failure data are available on a short and recent time window even if the systems have been implemented decades ago. The data could be assumed to correspond to the system stable regime, but one needs appropriate tools to take into account the lack of information on the system history. In this framework, it is of essential interest to infer the system's behavior in order to improve the maintenance policy and to plan efficient maintenance operations. Since the ARA_∞ model exhibits a convergence property (existence of a stable regime), it is a suitable candidate to model the data set.

In fact the ARA models, in particular ARA_1 and ARA_∞ have been intensively studied and in this framework many preventive maintenance policies are proposed. Nevertheless, in the literature, the imperfect preventive maintenance in an infinite horizon is rarely addressed. For instance

- Dagpunar (1998) proposes block replacement policy with ARA_∞ corrective maintenance and perfect preventive maintenance. The optimal duration between two preventive maintenance is derived, based on the computation of mean number of corrective maintenance actions during this period.
- Kijima et al. (1988) study block replacement policy with ARA_1 corrective maintenance and perfect preventive maintenance. They develop an approach to compute the mean number of ARA_1 corrective maintenance between two consecutive perfect preventive maintenance actions.
- Gilardoni, de Toledo, Freitas, and Colosimo (2015) study periodic preventive maintenance policy using ARA_1corrective maintenance and perfect preventive maintenance. The policy is first built for finite horizon and then extended to an infinite horizon.
- Tsai, Liu, and Lio (2011) study planned preventive maintenance policy. The preventive maintenance effect is similar to the ARA_1 maintenance, whereas corrective maintenance is minimal. The policy is optimized in a finite horizon.
- Gilardoni and Colosimo (2007) propose a similar policy (ARA_1 preventive maintenance and minimal corrective maintenance) optimized on an infinite horizon.

The main contribution of our paper is first to derive original theoretical properties of the $WARA_\infty$ model and then to develop original preventive maintenance policies. Regarding specifically the optimization of the preventive maintenance policy, our main contributions are listed as follows:

- For the first time, maintenance policies are derived on an infinite horizon considering imperfect CM and imperfect PM. Therefore there is at no moment an as good as new (perfect, renewal) replacement of the system during its operational lifetime. All the papers in the literature consider renewals under an infinite horizon.
- Thanks to the theoretical results in the stationary regime from the first Section, efficient and analytical approximations of the optimal maintenance policy are obtained without using Monte Carlo simulations.

The remainder of the paper is organized as follows. In Section 2, properties of the $WARA_\infty$ are developed to obtain statistical distributions of interest in both the transient and the stationary regimes. Based on these distributions, planned preventive maintenance policies are proposed in Section 3, along with numerical illustrations.

2. Properties of the $WARA_\infty$ model

2.1. The repair process

A repairable system has been observed since it was new. The observations consist of the successive maintenance times $\{T_i\}_{i\geq 0}$. The corresponding inter-maintenance times are denoted $\{X_i\}_{i\geq 1}$ and the repair process can also be characterized by a counting process $\{N_t\}_{t\geq 0}$ where $N_t = \sum_{i=1}^{\infty} \mathbb{1}_{\{T_i<t\}}$. By convention, T_0 and X_0 are equal to zero and time can be either calendar or operational. The distributions are obtained from the failure intensity defined in (1), where $\mathcal{H}_{t^-}$ is the history of the repair process at time t^-, commonly the failure times before t.

$$\forall t \geq 0,\ \lambda_t = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(N_{t+\Delta t} - N_{t^-} = 1 | \mathcal{H}_{t^-}) \tag{1}$$

The failure rate of a new system, called *initial failure intensity* and denoted $\lambda(t)$, is assumed to be a deterministic and continuous function of time. It corresponds to the hazard rate of T_1. The cumulative hazard rate function is denoted $\Lambda(t) = \int_0^t \lambda(u)du$. f, F and R are the corresponding probability density function, cumulative distribution function and survival function, respectively. As industrial systems are assumed to wear out, the initial failure intensity is traditionally increasing. Consequently, the two-parameter Weibull distribution has been chosen as in (2). For wearing-out systems, the shape parameter β is greater than 1.

$$\forall t \geq 0,\ \lambda(t) = \alpha\beta t^{\beta-1} \tag{2}$$

A virtual age model (Kijima et al., 1988) assumes that after the *ith* repair, the system behaves as a new and unmaintained one of age A_i. This age is called *effective age*. The assumption is mathematically described in (3), where Z is the time to failure of a new system and has the same distrib ution as X_1.

$$\forall i \geq 0, \forall t \geq 0,\ P(X_{i+1} > t | X_1, \ldots, X_i) = P(Z > A_i + t | Z > A_i) \tag{3}$$

The conditional survival function of the $(i+1)th$ inter-failure time in (3) is simply $R(A_i + t)/R(A_i)$. The age of the system A_0 at the beginning of the observation is zero if the system is as good as new, and greater than 0 otherwise. At a given time t, the virtual age of the system V_t is obtained from the latest effective age and the elapsed time since the last repair as in (4).

$$V_t = A_{N_{t^-}} + t - T_{N_{t^-}} \tag{4}$$

The virtual age of the system just before the *ith* repair is denoted A_i^-. The failure intensity can be derived from the initial failure intensity as in (5). The variation of the virtual age V_t and of the chronological time are identical between two consecutive failures.