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Using discrete-time mathematical programming to optimise the extraction rate of a durable non-renewable resource with a single primary supplier

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A B S T R A C T

A non-linear discrete-time mathematical program model is proposed to determining the optimal extraction policy for a single primary supplier of a durable non-renewable resource, such as gemstones or some metals. Karush, Kuhn and Tucker conditions allow obtaining analytic solutions and general properties of them in some specific settings. Moreover, provided that the objective function (i.e., the discounted value of the incomes throughout the planning horizon) is concave, the model can be easily solved, even using standard commercial solver. However, the analysis of the solutions obtained for different assumptions of the values of the parameters show that the optimal extraction policies and the corresponding prices do not exhibit a general shape.

1. Introduction

This paper deals with the optimal production policy for a single owner of the primary source of a durable non-renewable (therefore, exhaustible) resource, such as gold or diamonds. The problem is approached by means of non-linear discrete-time mathematical programming, what allows, under very general assumptions and by means of Karush, Kuhn and Tucker conditions, obtaining analytic solutions and general properties of them in some particular settings and computing easily the optimal policies.

Although most economic theory is not explicit about whether inputs into production are renewable or non-renewable, this distinction has significant implications of the optimal policies of producing and pricing the resource.

Natural resources can be renewable (e.g. fish stocks or forests) or non-renewable (all minerals). Among the latter, some (gemstones, precious metals and other metals like copper) are durable, whereas others (e.g. all kinds of fossil fuels, phosphates and other mineral fertilizers, and fossil water) are not. Non-durable resources disappear as such when they are used (burnt or dispersed), while durable resources may be reused, perhaps after recycling.

Therefore, when a non-renewable resource is durable, at any time there is an inventory of the resource in the ground and an inventory of the already used amounts of the resource that are potentially reusable.

Since the seminal papers by Gray [6] and Hotelling [7], where the famous Hotelling's rule concerning the price evolution of an exhaustible resource in a competitive market is stated, a certain number of papers and books on the economics of non-renewable resources have been published. The great majority of these publications (many relevant references can be found in [5]), explicitly or not, deal exclusively with non-durable resources, while the literature on the economics of durable non-renewable resources is relatively scarce. From the ten references included in a recent paper on this subject that deals with the prices of durable exhaustible resources under stochastic investment opportunities [1] only one [10] is specifically devoted to durable non-renewable resources. Hence, this topic remains largely unexplored.

In any case, as it will be shown below, research on this issue has revolved mainly around the conditions under which the Hotelling rule is valid or it is not. However, the objective of the present paper is to show the use of a non-linear discrete-time mathematical programming model to find the optimal extraction policies of the single owner of a durable non-renewable resource in a variety of scenarios.

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The implications of the durability for a monopolist[1] were analysed in Coase [4], where was stated what later has been known as the Coase Conjecture, namely, that if a durable-goods monopolist were unable to precommit to a future sales trajectory, market power would disappeared “in the twinkling of an eye”. Karp [8,9] specifies several settings in which the Conjecture fails.

Some papers deal with durable renewable goods monopolies, considering the problems derived from the fact that the sale of their products creates a secondary market beyond the control of the monopolists, which lead to compare selling versus renting [2,11,13,16]. Although the possibility of renting, which arises in these settings, are hardly applicable to durable exhaustive resources, these questions are considered in Malueg and Solow [12,14] in spite of that those two papers deal with this kind of resources.

Other researches concerning the economics of durable non-renewable resources focus on the validity of Hotelling’s rule for this kind of resources, as it is shown below.

Stewart [15] uses an optimisation discrete-time model and the Lagrange multiplier technique (what implies the assumption that the resource has to be depleted unless it is unlimitedly available) to compare, regarding the production throughout a finite time horizon of a durable exhaustible resource, the strategy of a competitive extractive industry with that of a monopolistic one. The author considers a general demand function that may vary from period to period and the notion of quasi-durability, which is quantified by means of a coefficient corresponding to the fraction of the stock of the extracted resource that remains from one period to the next. Stewart concludes that Hotelling’s rule applies to competitive and monopolistic markets, although in these latter, contrarily to that happens in the former, the optimal strategy may lead to falling prices.

Levhari and Pindyck [10], a fundamental contribution on the subject, using a continuous time infinite horizon formulation with growing demand and the Maximum Principle, criticise Stewart’s conclusions and argue that, although in a competitive market the price minus the marginal cost will rise at the rate of interest, this does not imply that price is steadily rising. The authors also discuss briefly the case of monopolistic markets and conclude that this rule does not hold in them. Besides, they point out that the evolution of the prices of durable resources “have shown long secular declines during at least part of their history, and in many cases have indeed been U-shaped over the long term (50–100 years)” and show that, under specific assumptions, their models can explain these behaviours.

Chilton [3], however, show that, if a convenient definition of marginal revenue is used, Hotelling’s rule extends to the case of monopolistic extraction of a durable good.

Malueg and Solow [12] analyse in detail the two-periods case under the assumptions of monopoly, static linear demand function, and perfect durability. They adapt a model from Bulow [2], with the additional assumption that the resource is exhaustible. Their analysis focusses on the differences that exhaustibility induces in the monopoly equilibrium of durable resources.

The same authors [14] analyse if monopoly leads or not to overconservation in the case of durable exhaustive resources. They use two models with static linear demand functions and an infinite horizon (a discrete-time model with perfect durability and a continuous-time one in which costs are an increasing function of cumulative production) and obtain from them similar results, with the general conclusions that monopoly is overconservative and prices fall monotonically during the production period.

In the present paper, a discrete-time non-linear mathematical programming model is proposed for determining the optimal policies of the single primary supplier of a durable exhaustible resource, under a variety of assumptions. This approach allows dealing with any evolution of the demand function throughout the time, any number of periods of the planning horizon and either with perfect durability or any degree of partial durability. Moreover, it makes easier the computation of the optimal policies and permits also analysing the properties of these policies in diverse settings.

The structure of the rest of the paper is as follows. The adopted assumptions and the mathematical programming model are stated in Section 2. The properties of the optimal solutions in several particular settings are discussed in Section 3, which also contains numerical examples. Section 4 closes the paper with some concluding remarks and future research lines.

2. Assumptions and model formulation

We consider a finite planning horizon divided into T periods. The equilibrium price, p_t, for each period, t, is a function, φ_t, of the stock of resource in circulation, s_t; i.e. $p_t = \varphi_t(s_t)$.[2] At the beginning of the planning horizon, the single primary supplier possesses an amount R of the resource and the stock in circulation is s_0.

We assume that the costs of production and distribution are negligible, although they could be easily incorporated if they are constant or depending on time and not on the amount of resource in the hands of the monopolist.

The stock in circulation in any period, s_t, is assumed to be equal to $\rho \cdot s_{t-1} + x_t$, where $x_t\,(\geq 0)$ is the amount of resource extracted and introduced into the market by the monopolist in period t and $\rho\,(\in (0,\ 1])$ is the proportion of the stock available in $t-1$ that is still available in t (a part of the available stock can deteriorate, be dispersed or lost or not considered marketable by its owners). The value zero is excluded, because in this case the resource would be non-durable; $\rho = 1$ corresponds to a perfectly durable resource and $0 < \rho < 1$ to the infinitely many degrees of partial durability. Of course, the value of this parameter may depend on time; however, we assume, for the sake of simplicity of the formulations, that it does not (relaxing this assumption, on the other hand, is straightforward).

Let α_t $(t = 1, \ldots, T)$ be the discount factor corresponding to period t .

Then, the policy that maximises the present value of the single supplier can be determined by means of solving the following mathematical program:

Model MODER

(Monopolistic Optimisation for a Durable Exhaustible Resource)

$$\text{maximise } z = \sum_{t=1}^{T} \alpha_t \cdot \varphi_t(s_t) \cdot x_t$$
$$= \sum_{t=1}^{T} \alpha_t \cdot \varphi_t\left(\rho^t \cdot s_0 + \sum_{\tau=1}^{t} \rho^{t-\tau} \cdot x_\tau\right) \cdot x_t$$

[1] Note that to speak of monopoly in relation to a durable good can be considered to a certain extent as an abuse of language, since the supply can come from the possessor of the primary source of the good and from any of its holders. However, for the sake of simplicity, as many authors do, the terms monopoly and monopolistic are used in the present paper in this specific sense.

[2] Although some authors (e.g. [15]) refer the equivalents of φ_t function as the inverse demand functions, others [10] avoid the use of this denomination, as we do in the present paper (except when describing the work of authors that use it). Note that, strictly speaking, φ_t is the relation between the stock of resource in circulation and the price and that the stock in circulation does not necessarily coincides with the supply of the resource, in the sense of the amount put to sale.