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Proof Of Normal Subgroups and Factor Groups Based on Java Programming Language

Ngarap Im Manik [a]

[a] Mathematics Departement, School of Computer Science, Bina Nusantara University
Jln. Kebon Jeruk Raya No.27, Jakarta 11480, Indonesia

Abstract

This research succeeded in developing an application program based on Java programming language. The software is able to prove abstract algebra, especially Normal Subgroups and Factor Groups. The result from the software program developed indicated that the proof of Normal Subgroups and Factor Groups could be done properly and correctly and relatively faster than if done manually.

* Correspondent Author : Ngarap Im Manik
Email: ngarap.imanuel@yahoo.com

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1. Introduction

Abstract algebra, or also known as mathematical structure, is a branch of mathematics that deals with the study of quantity, relationship, and structures. More specifically, abstract algebra is a study of algebraic structures, such as groups, rings, and fields. Abstract algebra is growing rapidly because the implementation of the characteristics of algebraic structures is much helpful in the development of problem-solving methods which are abstract and difficult to be represented by usual algebraic operations.[1]

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Some research topics were done before. Caenepel and Verschoren (2014) [2] discussed noncommutative rings through a group. Then Wallace (2014)[3] proved algebraic structure of rings by utilizing group theorem. Similarly, Okur et al. (2011)[4] developed the GAP (Group, Algorithm, Programming) model to prove groups and subgroups. Moreover, Manik et al[5] made a software design to prove specific groups (groups, subgroups, and homomorphism groups) using a computer. Furthermore, Iswati and Suryanto (2010)[6] discussed algebraic structures built on a group so that the properties that apply to the group will also apply to the K-Algebra. If there are subgroups and homomorphism groups, then in K-Algebra there are K-Subalgebra & K-Homomorphisms.

In Nancy (2009)[7], the application program includes only common algebraic structures to Abelian group (commutative). In addition, the previous application program supported only on testing an algebraic system. Then Saputra (2010)[8] designed a program that is able to test specific group forms, namely normal subgroups and factor groups.

Based on the previous researches mentioned, this research distinguishes on the testing scope. It was designed to be able to prove normal subgroups and factor groups. In addition, user interface was also re-designed, so it is easier for user navigation to switch between modules in the program, user-friendly, and efficient. Another thing is the application program adds explanations of the theory for learning, so that user can comprehend about what has been proved theoretically.

2. Methods

2.1 Normal Subgroups and Factor Groups

Definition Let H be a subgroup of the group G, and let $a \in G$. The set $aH = \{ x \in G \mid x = ah \text{ for some } h \in H \}$ is called the **left coset** of H in G determined by a. Similarly, the **right coset** of H in G determined by a is the set $Ha = \{ x \in G \mid x = ha \text{ for some } h \in H \}$.
The number of left cosets of H in G is called the **index** of H in G, and is denoted by [G:H].

Proposition 1. Let H be a subgroup of the group G, and let a,b be elements of G. Then the following conditions are equivalent:
(1) $bH = aH$; **(2)** $bH \subseteq aH$; **(3)** $b \in aH$; **(4)** $a^{-1}b \in H$.
A result similar to Proposition 1 holds for right cosets. Let H be a subgroup of the group G, and let $a,b \in G$. Then the following conditions are equivalent:
(1) $Ha = Hb$; (2) $Ha \subseteq Hb$; (3) $a \in Hb$; (4) $ab^{-1} \in H$; (5) $ba^{-1} \in H$; (6) $b \in Ha$; (7) $Hb \subseteq Ha$.

The index of H in G could also be defined as the number of right cosets of H in G, since there is a one-to-one correspondence between left cosets and right cosets [9].

Definition A subgroup H of the group G is called a **normal** subgroup if $ghg^{-1} \in H$ for all $h \in H$ and $g \in G$.

Proposition 2. Let H be a subgroup of the group G. The following conditions are equivalent:
- **(1)** H is a normal subgroup of G;
- **(2)** $aH = Ha$ for all $a \in G$;
- **(3)** for all $a,b \in G$, abH is the set theoretic product (aH)(bH);
- **(4)** for all $a,b \in G$, $ab^{-1} \in H$ if and only if $a^{-1}b \in H$.

Example Any subgroup of index 2 is normal.

Factor groups

Proposition 3. Let N be a normal subgroup of G, and let $a,b,c,d \in G$. If $aN = cN$ and $bN = dN$, then $abN = cdN$.