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Impact of selection methods on the diversity of many-objective Pareto set approximations

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Abstract

Selection methods are a key component of all multi-objective and, consequently, many-objective optimisation evolutionary algorithms. They must perform two main tasks simultaneously. First of all, they must select individuals that are as close as possible to the Pareto optimal front (convergence). Second, but not less important, they must help the evolutionary approach to provide a diverse population. In this paper, we carry out a comprehensive analysis of state-of-the-art selection methods with different features aimed to determine the impact that this component has on the diversity preserved by well-known multi-objective optimisers when dealing with many-objective problems. The algorithms considered herein, which incorporate Pareto-based and indicator-based selection schemes, are analysed through their application to the Walking Fish Group (WFG) test suite taking into account an increasing number of objective functions. Algorithmic approaches are assessed via a set of performance indicators specifically proposed for measuring the diversity of a solution set, such as the Diversity Measure and the Diversity Comparison Indicator. Hypervolume, which measures convergence in addition to diversity, is also used for comparison purposes. The experimental evaluation points out that the reference-point-based selection scheme of the Non-dominated Sorting Genetic Algorithm III (NSGA-III) and a modified version of the Non-dominated Sorting Genetic Algorithm II (NSGA-II), where the crowding distance is replaced by the Euclidean distance, yield the best results.

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1. Introduction

Multi-objective Optimisation Problems (MOPs) are those problems where several conflicting objective functions must be optimised simultaneously. MOPs with more than three objective functions are usually known as *Many-objective Optimisation Problems* (MaOPs) in the related literature [1]. The solution to MOPs and, consequently, MaOPs

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is a set of trade-off points referred to as the *Pareto optimal front* or *Pareto optimal set*. If a particular MOP satisfies a set of requirements, e. g., linearity or convexity of the objective functions or convexity of the feasible set, then the Pareto optimal set can be determined by mathematical programming approaches[2]. In the general case however, finding the solution of a MOP is an *NP*–complete problem[3]. As a result, heuristic or meta-heuristic methods, such as *Multi-objective Evolutionary Algorithms* (MOEAs)[4], have arisen as one of the most popular techniques to successfully address MOPs.

One key component of MOEAs is the selection scheme, i.e., the mechanism used to select individuals that will survive, and which is responsible for both convergence and diversity of the solution set provided. A significant number of MOEAs incorporates a Pareto-based selection scheme, which usually considers two separate selection criteria[5]. First, individuals are ranked by applying Pareto optimality, thus giving preference to those individuals that are non-dominated. Second, a diversity-based selection criterion is also applied to distinguish individuals belonging to the same rank. Although Pareto-based MOEAs have proven to be successful optimisers for a wide variety of MOPs, it has been recently demonstrated that Pareto-based selection is not suitable for MaOPs. One of the main reasons is that the number of non-dominated individuals exponentially increases with the number of objective functions. In this scenario, the selection scheme becomes inaccurate when ranking individuals and the selection pressure of the whole MOEA diminishes. Hence, individuals selected to survive may not be close enough to the Pareto optimal front due to the lack of convergence of the approach.

Two main paths have been explored in order to improve the performance of Pareto-based MOEAs when tackling MaOPs[5]. The first one is to propose novel definitions of dominance, such as dominance area control[6] and L-optimality[7], which allow the selection pressure of the MOEA to be increased. The second option focuses on improving or replacing the diversity-based selection criterion. In addition to the aforementioned options, different types of MOEAs have been proposed for solving MaOPs as an alternative to Pareto-based MOEAs[5]: *decomposition-based* MOEAs, *grid-based* MOEAs and *indicator-based* MOEAs.

Bearing all the above in mind, the main contribution of the current work is a comprehensive study about the impact that selection mechanisms have on the diversity preserved by MOEAs when dealing with MaOPs. For doing that, well-known MOEAs, which incorporate selection mechanisms with different features, are applied to MaOPs with a scalable number of objectives belonging to the *Walking Fish Group* (WFG) test suite[8]. The way in which the performance of MOEAs can be measured has arisen as an important research area[9]. As a result, a considerable number of quality indicators, like the *hypervolume indicator*[10] or the *Diversity Comparison Indicator*[9], has been proposed for measuring either convergence or diversity or both of them. In this work, we will focus on quality indicators specifically designed for measuring the diversity of a solution set.

The rest of this paper is organised as follows. Section 2 is devoted to describe all the foundations related to the work carried out herein, including the formal definition of a MOP, the particular MOEAs we have considered for our study, as well as their selection mechanisms. The quality indicators selected to evaluate the diversity of the solution sets provided by those MOEAs are also introduced. Then, the computational experiments performed, as well as the results obtained, are shared in Section 3. Finally, Section 4 gives some conclusions and lines of future work.

2. Foundations

A *Multi-objective Optimisation Problem* (MOP) can be defined as the problem in which a set of *objective functions* $f_1(\boldsymbol{x}), \ldots, f_M(\boldsymbol{x})$ should be jointly optimised;

$$\min \ \boldsymbol{F}(\boldsymbol{x}) = \langle f_1(\boldsymbol{x}), \ldots, f_M(\boldsymbol{x}) \rangle \, ; \ \boldsymbol{x} \in \mathcal{S} \, ; \tag{1}$$

where $\mathcal{S} \subseteq \mathbb{R}^n$ is known as the *feasible set* and can be expressed as a set of restrictions over the *decision set*, in our case, $\mathbb{R}^n$. The *image set* of $\mathcal{S}$ produced by function vector $\boldsymbol{F}(\cdot)$, i.e., $\mathcal{O} \subseteq \mathbb{R}^M$ is called the *feasible objective set* or *criterion set*. The solution to these types of problems is a set of trade-off points. The optimality of a given solution can be defined in terms of the Pareto dominance relation.

Definition 1 (Pareto dominance relation)**.** *For the optimisation problem specified in (1) and having $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{S}$, $\boldsymbol{x}$ is said to dominate $\boldsymbol{y}$ (expressed as $\boldsymbol{x} \prec \boldsymbol{y}$) iff $\forall f_j$, $f_j(\boldsymbol{x}) \leq f_j(\boldsymbol{y})$ and $\exists f_i$ such that $f_i(\boldsymbol{x}) < f_i(\boldsymbol{y})$.*