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ScienceDirect
Applied Soft Computing 8 (2008) 1232–1242
Applied Soft Computing
www.elsevier.com/locate/asoc

Review

On the absence of strict boundaries—Vagueness, haziness, and fuzziness in philosophy, science, and medicine☆

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Received 21 February 2007; accepted 28 February 2007
Available online 22 October 2007

Abstract

This contribution deals with developments in the history of philosophy, logic, and mathematics during the time before and up to the beginning of fuzzy logic. Even though the term ''fuzzy'' was introduced by Lotfi A. Zadeh in 1964/1965, it should be noted that older concepts of ''vagueness'' and ''haziness'' had previously been discussed in philosophy, logic, mathematics, applied sciences, and medicine. This paper delineates some specific paths through the history of the use of these ''loose concepts''. Vagueness was avidly discussed in the fields of logic and philosophy during the first decades of the 20th century—particularly in Vienna, at Cambridge and in Warsaw and Lvov. An interesting sequel to these developments can be seen in the work of the Polish physician and medical philosopher Ludwik Fleck.

Haziness and fuzziness were concepts of interest in mathematics and engineering during the second half of the 1900s. The logico-philosophical history presented here covers the work of Bertrand Russell, Max Black, and others. The mathematical–technical history deals with the theories founded by Karl Menger and Lotfi Zadeh. Menger's concepts of probabilistic metrics, hazy sets (ensembles flous) and micro-geometry as well as Zadeh's theory of fuzzy sets paved the way for the establishment of soft computing methods using vague concepts that connote the nonexistence of sharp boundaries.

Keywords: Vagueness; Haziness; Fuzziness; Philosophy of science; Logic; Language; Medicine; Fuzzy sets; Micro-geometry

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☆ This article is a modified and revised version of my conference contributions to the IFSA 2005 World Congress [1] and the BISC Special Event'05, Forging New Frontiers. 40th of Fuzzy Pioneers (1965–2005) in Honor of Prof. Lotfi A. Zadeh [2]. A version for researchers with specific philosophical background is available in my contribution to the book *Fuzzy Logic and Soft Computing Pioneers*, in the chapter entitled ''Pioneers of vagueness, haziness, and fuzziness in the 20th century'' [3]. My historical reconstruction of the genesis and development of the theory of fuzzy sets and systems has been published in German. An English version of this book appeared in 2007 in the Springer Series, *Studies in Fuzziness and Soft Computing* [4].

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1568-4946/$ – see front matter
doi:10.1016/j.asoc.2007.02.017

1. Introduction

Exact concepts, i.e. concepts with strict boundaries, bivalent logic that enables us to decide yes or no, mathematical formulations to represent sharp values of quantities, measurements and other terms are tools of logic and mathematics that have given modern science its exactness.

‘‘Vagueness’’ is also part of the vocabulary of modern science but ‘‘vague’’ did not become a technical term in philosophy and logic during the 18th and 19th century. In the 20th century, however, philosophers like Gottlob Frege, Bertrand Russell, Max Black, and others focused attention on and analyzed the problem of ‘‘vagueness’’ in modern science.

A separate and isolated development took place at the Lvov–Warsaw School of logicians. Their important contributions to modern logic were recognized when Alfred Tarski gave a lecture to the Vienna Circle in September 1929—following an invitation extended after the Viennese mathematician Karl Menger had got to know the Lvov–Warsaw scholars during his travels to Warsaw the previous summer. It turned out that these thinkers had been influenced by Frege’s studies. This was especially true of Tadeusz Kortabiński, who argued that a concept for a property is vague (Polish: chwiejne) if the property may be the case by grades [5], and Kazimierz Ajdukiewicz, who stated the definition that ‘‘a term is vague if and only if its use in a decidable context ... will make the context undecidable in virtue of those [language] rules’’ [6]. The Polish characterization of ‘‘vagueness’’ was therefore the existence of fluid boundaries [5,6].

In the first third of the 20th century there were several groups of European scientists and philosophers who concerned themselves with the interrelationships between logic, science, and the real world, e.g. in Berlin, Cambridge, Warsaw, and the so-called Vienna Circle. The scholars in Vienna regularly debated these issues over a period of years until the annexation of Austria by Nazi Germany in 1938 marked the end of the group. One member of the Vienna Circle was Karl Menger, who later became a professor of mathematics in the USA. As a young man in Vienna, Menger raised a number of important questions that culminated in the so-called principle of logical tolerance. In addition, in his work after 1940 on the probabilistic or statistical generalization of metric space, he introduced the new concepts ‘‘hazy sets’’ (ensembles flous), t-norms and t-conorms, which are also used today in the mathematical treatment of problems of vagueness in the theory of fuzzy sets.

This new mathematical theory to deal with vagueness was established in the mid 1960s by Lotfi A. Zadeh, who was then a professor of electrical engineering at Berkeley. In 1962 he described the basic necessity of a new scientific tool to handle very large and complex systems in the real world: ‘‘we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not describable in terms of probability distributions. Indeed, the need for such mathematics is becoming increasingly apparent even in the realm of inanimate systems, for in most practical cases the *a priori* data as well as the criteria by which the performance of a man-made system are judged are far from being precisely specified or having accurately-known probability distributions’’ ([7], p. 857). In the two years following the publication of this paper, Zadeh developed the theory of fuzzy sets [8–10], and it has been possible to reconstruct the history of this process [4].

Very little is known about the connectivity between the philosophical work on vagueness and the mathematical theories of hazy sets and fuzzy sets (see Fig. 1). In this article the author shows that there is common ground in the scientific developments that have taken place in these different disciplines, namely, the attempt to find a way to develop scientific methods that correspond to human perception and knowledge, which is not the case with the exactness of modern science.

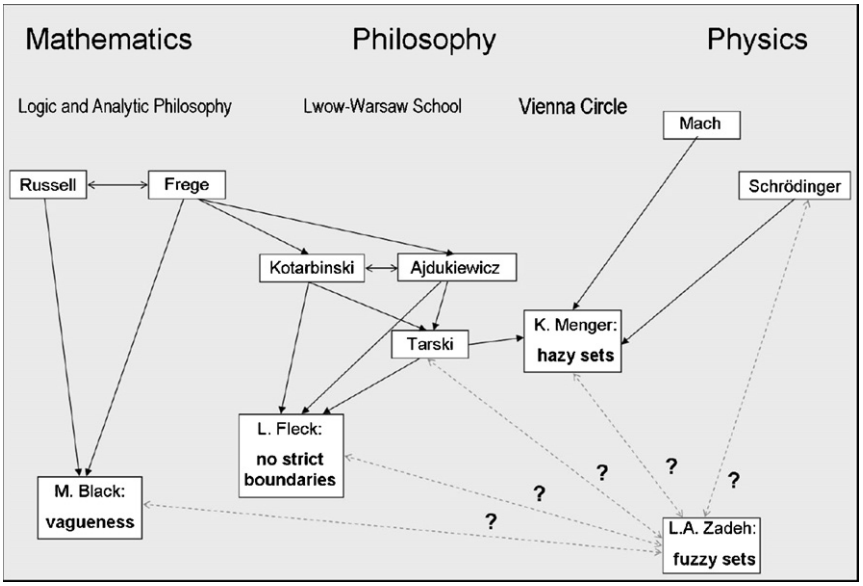

Fig. 1. Connectivities between the philosophical work on vagueness and the mathematical theories of hazy sets and fuzzy sets. Missing or unknown relations are dashed.