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Spectral Modes of Network Dynamics Reveal Increased Informational Complexity Near Criticality

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Abstract

What does the informational complexity of dynamical networked systems tell us about intrinsic mechanisms and functions of these complex systems? Recent complexity measures such as integrated information have sought to operationalize this problem taking a whole-versus-parts perspective, wherein one explicitly computes the amount of information generated by a network as a whole over and above that generated by the sum of its parts during state transitions. While several numerical schemes for estimating network integrated information exist, it is instructive to pursue an analytic approach that computes integrated information as a function of network weights. Our formulation of integrated information uses a Kullback-Leibler divergence between the multi-variate distribution on the set of network states versus the corresponding factorized distribution over its parts. Implementing stochastic Gaussian dynamics, we perform computations for several prototypical network topologies. Our findings show increased informational complexity near criticality, which remains consistent across network topologies. Spectral decomposition of the system's dynamics reveals how informational complexity is governed by eigenmodes of both, the network's covariance and adjacency matrices. We find that as the dynamics of the system approach criticality, high integrated information is exclusively driven by the eigenmode corresponding to the leading eigenvalue of the covariance matrix, while subleading modes get suppressed. The implication of this result is that it might be favorable for complex dynamical networked systems such as the human brain or communication systems to operate near criticality so that efficient information integration might be achieved.

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1 Introduction

Quantifying informational processes of dynamical networked systems has been increasingly useful as a unique window for probing internal system states and mechanisms that underlie

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observed phenomenological behaviors of many complex systems. Mapping structure-function relationships in this way by using information theory has paid off for studying both, complex biological systems such as the brain or engineered systems such as communication networks. A prominent information-theoretic complexity measure that has shown a recent resurgence of interest in the wake of consciousness research is integrated information (often denoted as Φ). It was first introduced in neuroscience as a complexity measure for neural networks, and touted as a correlate of consciousness [19]. Integrated information Φ is loosely defined as the quantity of information generated by a network as a whole, due to its causal dynamical interactions, that is over and above the information generated independently by the disjoint sum of its parts. As a complexity measure, Φ seeks to operationalize the intuition that complexity arises from simultaneous integration and differentiation of the network's structure and dynamics. Integration results in distributed coordination among nodes, while differentiation leads to functional specializations, thus enabling the emergence of the system's collective states. The interplay between integration and differentiation thus generates information that is highly diversified yet integrated, creating patterns of high complexity. Following initial proposals [17], [18], [19], several approaches have been developed to compute integrated information [1], [4], [5], [6], [8], [9], [11], [13], [14] (see also [2], [15], [16], [20] for other related measures). Some of these were constructed for networks with discrete states, others for continuous state variables. In this paper, we will consider stochastic network dynamics with continuous state variables because this class of networks model many biological as well as communication systems that generate multivariate time-series signals. We want to study the precise analytic relationship between the information integrated by these networks and the couplings that parametrize their structure and dynamics. It turns out that tuning the dynamical operating point of a network near the edge of criticality leads to a high rate of network information integration and that remains consistent across network topologies. To explain this phenomenon, we analyze the spectrum of the network's dynamics. This reveals that integrated information is coupled to the characteristics of the eigenmodes of the system's covariance matrix, and in turn these are related to the eigenvalues of the network's adjacency matrix. In this paper, we make these relationships precise.

2 Methods

We consider complex networks with linear multivariate dynamics and Gaussian noise. It follows that the state of each node is given by a random variable pertaining to a Gaussian distribution. For many realistic applications, Gaussian-distributed variables are fairly reasonable abstractions. The state of the network $\mathbf{X_t}$ at time t is taken as a multivariate Gaussian variable with distribution $\mathbf{P}_{\mathbf{X_t}}(\mathbf{x_t})$. $\mathbf{x_t}$ denotes an instantiation of $\mathbf{X_t}$ with components x_t^i (i going from 1 to n, n being the number of nodes). When the network makes a transition from an initial state $\mathbf{X_0}$ to a state $\mathbf{X_1}$ at time $t = 1$, observing the final state generates information about the system's initial state. The information generated equals the reduction in uncertainty regarding the initial state $\mathbf{X_0}$. This is given by the conditional entropy $\mathbf{H}(\mathbf{X_0}|\mathbf{X_1})$. In order to extract that part of the information generated by the system as a whole, over and above that generated individually by its irreducible parts, one computes the relative conditional entropy given by the Kullback-Leibler divergence of the conditional distribution $\mathbf{P}_{\mathbf{X_0}|\mathbf{X_1}=\mathbf{x}'}(\mathbf{x})$ of the whole system with respect to the joint conditional distributions $\prod_{k=1}^{n} \mathbf{P}_{\mathbf{M_0^k}|\mathbf{M_1^k}=\mathbf{m}'}$ of its irreducible parts [6]. Here $\mathbf{M_t^k}$ denotes the state variable of the k^{th}-component of the partitioned system at time t.