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Procedia Computer Science 108C (2017) 798–807

Procedia Computer Science

International Conference on Computational Science, ICCS 2017, 12-14 June 2017, Zurich, Switzerland

Quadrature blending for isogeometric analysis

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Abstract

We use blended quadrature rules to reduce the phase error of isogeometric analysis discretizations. To explain the observed behavior and quantify the approximation errors, we use the generalized Pythagorean eigenvalue error theorem to account for quadrature errors on the resulting weak forms [28]. The proposed blended techniques improve the spectral accuracy of isogeometric analysis on uniform and non-uniform meshes for different polynomial orders and continuity of the basis functions. The convergence rate of the optimally blended schemes is increased by two orders with respect to the case when standard quadratures are applied. Our technique can be applied to arbitrary high-order isogeometric elements.

Peer-review under responsibility of the scientific committee of the International Conference on Computational Science

Keywords: Isogeometric analysis, Finite elements, Numerical methods, Quadratures

1 Introduction

The development and use of isogeometric analysis (IGA) for partial differential equations (PDE) has continuously grown over the last decade [7, 8, 15, 16, 19, 20, 22]. Isogeometric analysis can use NURBS (Non-Uniform Rational B-Splines) or T-splines [9, 14, 27]. Thus, IGA uses the framework of computer aided design (CAD) systems to exactly represent complex geometries. This allows to simplify the process of mesh generation and refinement, as well as the information exchange between analysis and design. Isogeometric analysis has several attractive features. The basis functions can have higher continuity across element interfaces and hence the approximated solutions have global continuity of order up to $p-1$, where p is the order of the underlying polynomial. These highly continuous isogeometric solutions are more accurate and robust than standard finite element solutions per degree of freedom, although are more costly per degree of freedom [11–13, 26]. Several efficient implementations of the isogeometric analysis techniques in open source software are available [17, 18, 25, 29].

Dispersion analysis quantifies the approximation errors of a numerical method. The dispersive properties of various numerical methods, such as the finite element method (FEM), the spectral element method (SEM), among others, have been studied in detail in the last decades

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Peer-review under responsibility of the scientific committee of the International Conference on Computational Science
10.1016/j.procs.2017.05.143

[1, 21]. An accurate and cost-effective scheme can be obtained by employing a weighted average of these methods [24]. In particular, Ainsworth has shown that the optimal blending of FEM and SEM provides two orders of extra accuracy in the phase error [1, 2].

Herein, we describe the errors in the discrete approximations to elliptic eigenvalue problems for the normal modes and frequencies of free structural vibrations. A similar analysis can be applied to the Helmholtz equation (e.g., frequency domain wave propagation). We consider both the eigenvalue and eigenfunction errors. That is, we conduct a global error analysis by describing the errors in the eigenvalues and the eigenfunctions for all modes. The sum of the eigenvalue error and the square of the L_2 eigenfunction error equals the square of the error in the energy norm. To quantify the approximation errors for the case underintegration, we have generalized Strang's Pythagorean theorem to include the effect of inexact integration [10, 28].

2 Problem statement

We first analyse the Laplace eigenvalue problem in one dimension

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2} = \lambda u, \quad \text{for } x \in \Omega =]0,1[\\ &u(0) = u(1) = 0, \end{aligned} \tag{1}$$

with homogeneous Dirichlet boundary conditions. The following standard results are specialized for the one-dimensional case to simplify the discussion [10, 22, 28, 30]. The problem has a countable infinite set of eigenvalues $\lambda_j \in \mathrm{R}$ and an associated set of orthonormal eigenfunctions u_j (normalized with respect to the L_2 inner product)

$$0 \leq \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_j \leq ... \tag{2}$$

$$(u_j, u_k) = \int_\Omega u_j(x) u_k(x) dx = \delta_{jk}, \tag{3}$$

where δ_{jk} is the Kronecker delta which is equal to 1 when $i = j$ and 0 otherwise. These eigenfunctions form an L_2-orthonormal basis and are orthogonal also in the energy inner product

$$(\mathcal{L} u_j, u_k) = (\lambda_j u_j, u_k) = \lambda_j \delta_{jk}. \tag{4}$$

The standard weak form for the eigenvalue problem is stated as follows: Find all eigenvalues $\lambda_j \in \mathrm{R}$ and eigenfunctions $u_j \in V$ such that, for all $w \in V$,

$$a(w, u_j) = \lambda_j (w, u_j), \tag{5}$$

where

$$a(w, u_j) = \int_\Omega \frac{dw}{dx} \frac{du_j}{dx} dx, \tag{6}$$

and V is a closed subspace of $H^1(\Omega)$. We use the notation of [30] where $(\cdot, \cdot)$ and $a(\cdot, \cdot)$ are symmetric bilinear forms which define the following inner products

$$\|w\|_E^2 = a(w, w), \quad \|w\|^2 = (w, w), \tag{7}$$

for all $v, w \in V$. $H^1(\Omega)$ denotes the Sobolev space of functions

$$H^1(\Omega) = \{ f : \Omega \to R \mid \|f\|_{H^1} < \infty \}, \quad \|f\|_{H^1}^2 = \int_a^b \left[f^2(x) + \left(\frac{d}{dx} f(x) \right)^2 \right] dx. \tag{8}$$