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Asymptotically Local Synchronization of An Interdependent Network with Two Sub-Networks and Delays

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Abstract

In this article, the asymptotically local synchronization for a general interdependent dynamical network with two sub-networks and delays is investigated. The coupling function under consideration in this network is continuous and on behalf of both the intra-network and inter-network. Based on the Master Stability Function method, Linearization method and Lyapunov–Krasovskii Stability Theory combing with the Delay Differential Inequality technique, some elegant strategies are established which guarantee the asymptotically exponential local synchronization for each node in the network. Finally, several numerical simulations show the effectiveness and feasibility of the proposed schemes.

Keywords: Asymptotically exponential local synchronization; Interdependent network with delays; Master Stability Function; Lyapunov–Krasovskii stability theory ; Delay Differential Inequality

1. Introduction

Interdependent network is a kind of complex networks with sub-networks and also called a kind of networks of networks. There are two different kinds of links: connectivity links and dependency links, which is the fundamental property in the interdependent networks [1-2]. It will be significant for us to better design, organize, and maintain the future of our world if we knowing these links in the interdependent networks.

Because interdependent networks behave differently than single networks, interdependent networks have many different applications in many of the fields. In the early 2000s, there have been some scholars who classified, modelled, and analyzed the properties of interdependence between infrastructure systems, and yet evaluated the sudden or natural disaster vulnerability [3-5]. The literatures [6-7] considered the epidemics on interdependent networks. In some economic networks, which are composed of individuals, companies and banks, every node interacts with other nodes and then the whole network runs into a large scale cascading failures [8]. Interdependent

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networks have also been applied in transportation networks [9], climate [10], ecology [11] and physiological systems [12].

Similar with single networks, among all kinds of dynamical phenomena in the interdependent networks, synchronization is a very important one [13]. Although there are some literatures [14-17] which focused on synchronization of interdependent networks, the research is still at the initial stage. Based on the mean-field approximation, the literature [14] explored the synchronization in interdependent systems, where the one-dimensional network is ferromagnetically inter-coupled to the Watts-Strogatz small-world network. In [15], the authors showed how the algebraic connectivity experiences sharp transitions after the addition of sufficient links among interdependent networks. In [16], the generalized mutual synchronization between two controlled interdependent networks was investigated.

Inspired by the above discussion, this article investigates the asymptotically local synchronization for a general interdependent dynamical network with two sub-networks and delays. As far as I know, for this respect, there is no result in the existing literatures. And, it is still an open and challenging topic.

2. Model description and preliminary

For convenience, in this section, some models and preliminaries are introduced.

2.1 Model description

Considering the following delay interdependent complex network composed of two sub-networks and $N_1 + N_2$ coupled nodes, which is described by

$$\dot{x}_i^k(t) = f\left(x_i^k(t)\right) + d^k \sum_{j=1}^{N_k} a_{ij}^k H\left(x_j^k(t-\tau)\right) + \sum_{l=1}^{2} c^{kl}\left(H\left(x_i^l(t-\tau)\right) - H\left(x_i^k(t-\tau)\right)\right), i = 1, 2, \cdots, N_k, \quad k = 1, 2. \quad (1)$$

where $x_i^k \in R^n$ is the state vector of the node i in which the subscript stands for the node and the superscript for the sub-networks, $f\left(x_i^k(t)\right)$: $R^n \to R^n$ is a nonlinear smooth function which governs the dynamics of isolated node x_i^k, d^k represents the coupling strength of the k-th sub-network, $H : R^n \to R^n$ is a continuous function and on behalf of both the intra-network and inter-network coupling function, and τ is a delay time. $A^k = \left(a_{ij}^k\right) \in R^{N_k \times N_k}$ is the coupling weight configuration matrix of the k-th sub-network. If the node j connects the node i ($i \neq j$), $a_{ij}^k = 1$, otherwise $a_{ij}^k = 0$. The matrix A^k is diffusive if $a_{ii}^k = -\sum_{\substack{j=1 \\ j \neq i}}^{N_k} a_{ij}^k$, then $L^k = -d^k A^k$ is called a Laplacian matrix. If c^{kl} is the inter-networks coupling strength between two sub-networks, then $C = \left(c^{kl}\right) \in R^{2\times 2}$ is a coupling weight matrix between the k-th and the l-th sub-network which is also a negative Laplacian matrix if satisfying

$$c^{kk} = -\sum_{\substack{l=1 \\ l \neq k}}^{2} c^{kl}.$$

Hypothesis1. In this article, assuming that each node i in one sub-network depends on only one node j in the other sub-network (uniqueness condition), we further suppose that if node i in sub-network depends on the