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# Application of numerical time integration schemes to continuously variable transmission dynamics analysis

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**Abstract**
Numerical analysis of large-scale and multidisciplinary problems on high-performance computer (HPC) systems is one of the main computational challenges of the 21st century. Modelling the dynamics of a continuously variable transmission (CVT) is one of such problems. CVT has been used in the automotive industry for last decades. The complexity of physical interactions in CVT does not allow engineers to use any of the general purpose commercial software packages for CVT design. Our project aims to develop a mathematical model, computational algorithms and problem oriented in-house software for simulating CVT dynamics. One of the important steps in this project is the study of explicit and implicit time integration schemes for the stiff system of ordinary differential equations, as well as implementations of these schemes on HPC systems.

*Keywords:* large-scale models, time integration schemes, nonlinear problems, stiff systems

## 1 Introduction

We consider the specific design of CVT that includes two shafts (driving and driven) on elastic supports (Figure **1**a,b); two sheaves attached to each shaft – one being able to move along the shaft, and the other one being fixed; the chain consisting of rocker pins and plates (Figure **1**c). The ends of rocker pins of the chain are in contact with the sheaves, which are clamped with axial forces. Thus, axial positions of moving sheaves determine the configuration of chain and the gear ratio. Torque is transmitted due to friction forces that arise at contact points between pins and sheaves.

A detailed description of our mathematical model of CVT dynamics can be found in [1]. The model employs mathematical formalism of Lagrangian mechanics to describe the dynamics of a discrete system of deformable bodies: pins, plates, shafts and pulleys. The resulting system of nonlinear ordinary differential equations is then numerically integrated over time.

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The current implementation of our CVT simulation software employs the explicit time integration algorithm, namely the well-known Runge-Kutta 4th order (RK4) scheme, modified so that the pin-pulley contact state transitions occurring within a time step can be taken into account. While explicit schemes are most easily implemented, they have inherent properties that limit their applicability for stiff systems. Most importantly, the numerical stability of such schemes is determined by the product of the highest natural frequency of the system and the value of time integration step. Therefore, for stiff systems, very short time steps have to be used to ensure the stability; this makes explicit schemes inefficient and severely decreases simulation performance.

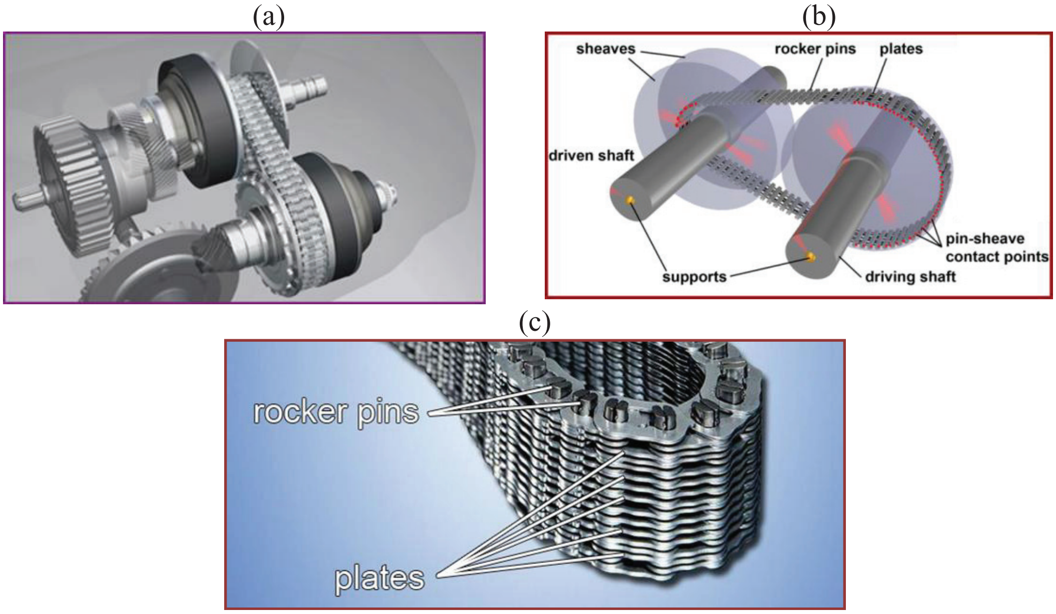


Figure 1. (a,b) Continuously variable transmission; (c) View of CVT chain in details: pins and connecting plates

For the analysis of stiff systems dynamics, the implicit time integration schemes turn out to be more efficient than the explicit schemes [2]. Therefore, we aim to investigate the application of some implicit methods suitable for stiff systems to our CVT dynamics model. In contrast to explicit schemes, the stability of implicit ones does not impose restrictions on the time integration step value. Much larger steps can be used with implicit schemes; the restriction in this case is imposed by the accuracy requirement and the ability to solve algebraic equations at each time step.

Implicit numerical schemes are more complex to implement than explicit schemes, because they require solving a system of algebraic equations at each time integration step. Efficiency of numerical solution of these equations determines the overall efficiency of an implicit scheme. Typically, any specific problem requires an individual selection and tuning of the algebraic equation solver.

# 2 Current investigation and future plans

Within this project, our goal is to investigate the application of two numerical integration schemes, Rosenbrock [2-3] and Newmark-beta [4], to our CVT dynamics model.