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Iterative Method for Solving a System of Linear Equations

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Abstract

The systems of linear equations are a classic section of numerical methods which was already known BC. It reached its highest peak around 1600-1700 due to the public demand for solutions of technical and engineering tasks, nevertheless, it is still topical nowadays. This paper describes another iterative approach to solving linear systems, which is based in the multiple transfers of the solution proximity point towards the solution itself, simultaneously reducing the differences of all the system equations.

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Keywords: The systems of linear equations; Iterative methods for solving systems of linear equations; The requirements for the matrix of the equation system

1. Introduction

A new demand for new means of solving systems of linear equations appeared at the same time as the computing technology emerged which promoted a rapid development of numerical methods for modelling physical processes by sampling (sub-dividing) the calculation range as well as replacing the differential operations by similar algebraic operations. According to the requirements of the final differences, final elements and their modifications, direct and iterative methods for approaching a poorly completed diagonal matrix with a strong main diagonal were developed[1, 2]. Methods for efficient storage of the equation system were developed, taking into account the symmetry of the

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matrix according to the main diagonal for both direct and iterative methods. In recent years, with the introduction of new numerical methods (super elements, the method of border elements), there has been a necessity for solving systems of linear equations with a completely filled matrix and one which does not possess the main diagonal dominance[3]. Iterative methods are often used for solving such tasks and the methods have been developed from the Gauss-Seidel method[4,5].

Solving systems of linear equations by iterative methods (such as Gauss-Seidel method) involves the correction of one searched-for unknown value in every step (see Fig. 1a) by reducing the difference of a single individual equation; moreover, other equations in this process are not used[5]. In order to accelerate the convergence of the iterative process, the methods are complemented by wellness principles which optimize the rate of the variable change in the iterative process.

The approach that is suggested in this article is based in every iteration simultaneously and is meant to reduce the differences of all the system equations by changing all the values of the unknown variables in the system (see Fig. 1b). The key question is how to organize the iterative process which is correcting the unknown variables and reducing the differences of all the equations at the same time.

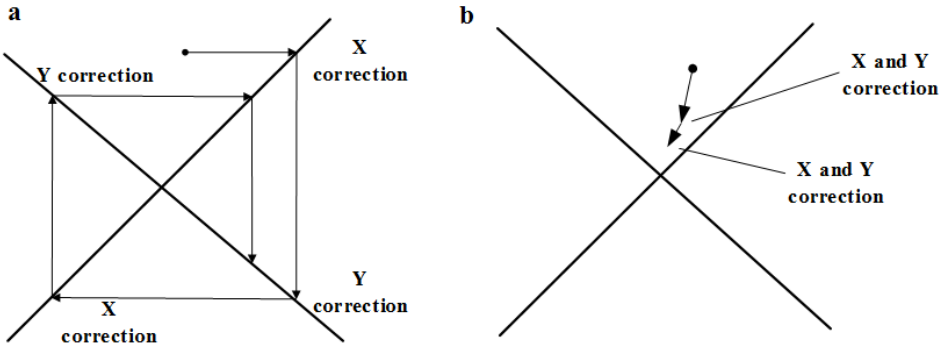

Fig 1. (a) the iterative solution according to the Gauss-Seidel method – a single variable is corrected in every step; (b) the provided iterative solution scheme – all the variables are corrected in every step.

Initially, the solution for the two equations system is to be viewed, and then it is to be generalized to any number of final dimensions.

2. The essence of the method for a two equations system

The initial solution approximation $P^0(x, y)$ is freely assumed. It can easily be shown that any point P(x, y) in the XY plane is located further from the solution of the equation system than the projection P(x, y) of equations (see Fig. 2). Subsequently, in order to solve the equation systems the following iterative process can be implemented in Fig. 2:

- The proximity P^i (x, y) projection on the equations are found
- The midpoint (or arithmetic mean) is assumed as the new proximity $P^{i+1}(x^{i+1}, y^{i+1})$ between the projections of the point $P^i(x^i, y^i)$ on the system equations a and b
- The correction of the proximity is iteratively repeated

In the case of solving iterative linear systems, point P(x,y) is moved step by step – $P^0(x^0,y^0)$, $P^1(x^1,y^1)^1$, $P^2(x^2,y^2)^2$, ... – until a certain state has been found in which the proximity of the solution is close enough to the solution R(x,y) of the systems of linear equations. Since the solution R(x,y) is sought for, the quality of the proximity has to be assessed, for instance, by applying the approximate solution in the equations and evaluating the differences, determining that the proximity no longer changes in the iterative process etc.