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The controllability of parabolic systems with delay and distributed parameters on the graph

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Abstract

The optimal boundary control problem for a differential system with delay and distributed parameters on the graph is considered. As a state-space systems and the space of boundary control and observation, the classes of integrable functions is used. The conditions of the unique solvability of the problem of optimal control are obtained. And finally, the existence conditions of a unique control action and the controllability system are obtained.

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1. Introduction

It is presented an approach based on a priori estimates of generalized solutions of the initial boundary value problems for equations of parabolic type with distributed parameters on the graph. On this basis, the problem of boundary control with delay is considered, conditions for the existence of a single control and relations that characterize this control are obtained. The questions of controllability of differential systems in Banach spaces, presented by the results of monograph J.L.

Lions which became the classic[1], significantly influenced at the choice of the author's way of the study of controllability of evolution systems, which is new in its formulation and have an interesting analogy with the study of

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the dynamics of multiphase flows of viscous media. The paper considers the evolution system, whose state is defined as a weak solution of the boundary value problem for a parabolic equation with distributed parameters on the graph and which describes techniques to set the property of controllability of the system. All consideration are used the limited an arbitrary connected directed graph, allowing the presence of cycles.

2. The basic concepts and propositions

Throughout the paper, we use the following generally accepted system of symbols[1]. Denote by $\partial\Gamma$ and $J(\Gamma)$ the sets of boundary ζ and internal ζ nodes, respectively, in a graph Γ. Let Γ_0 to be the union of all edges containing no terminal points; $\Gamma_T = \Gamma_0 \times (0,T)$ ($\Gamma_t = \Gamma_0 \times (0,t)$), $\partial\Gamma_T = \partial\Gamma \times (0,T)$ ($\partial\Gamma_t = \partial\Gamma \times (0,t)$). Each edge γ of the graph Γ is oriented, parameterized by $[0,1]$ and the variable $x \in [0,1]$.

Introduce necessary spaces: $L_p(\Gamma)$ ($p = 1,2$) means the Banach space of measurable on Γ_0 functions with finite norm $\| u \|_{L_p(\Gamma)} = (\int_\Gamma u^p(x)dx)^{1/p}$ (the spaces $L_p(\Gamma_T)$, $p = 1,2$ is defined similarly); $W_2^1(\Gamma)$ designates the space of functions from $L_2(\Gamma)$ possessing the first-order generalized derivative, which belongs to $L_2(\Gamma)$, the norm in $W_2^1(\Gamma)$ is established by the scalar product $(u,v)_{W_2^1(\Gamma)} = \int_\Gamma (u(x)v(x) + u'(x)v'(x)dx)dx$; $L_{2,1}(\Gamma_T)$ represents the space of functions from $L_1(\Gamma_T)$ having the norm $\| u \|_{L_{2,1}(\Gamma_T)} = \int_0^T (\int_\Gamma u^2(x,t)dx)^{1/2} dt$; $W_2^{1,0}(\Gamma_T)$ designates the space of functions $u(x,t) \in L_2(\Gamma_T)$ possessing the first-order generalized derivative with respect to x, which belongs to $L_2(\Gamma_T)$, $\| u \|^2_{W_2^{1,0}(\Gamma_T)} = \int_{\Gamma_T} (u(x,t)^2 + u_x(x,t)^2)dxdt$.

Further, let $V_2(\Gamma_T)$ means is the set of functions $u(x,t) \in W_2^{1,0}(\Gamma_T)$ with a finite norm

$$\| u \|_{2,\Gamma_T} = \max_{0 \le t \le T} \left\| u(x,t) \right\|_{L_2(\Gamma)} + \left\| u_x \right\|_{L_2(\Gamma_T)}, \tag{1}$$

which strongly continuous in t in the norm $L_2(\Gamma)$.

Consider the bilinear form $\ell(\mu,\nu) = \int_\Gamma \left(a(x) \frac{d\mu(x)}{dx} \frac{d\nu(x)}{dx} + b(x)\mu(x)\nu(x) \right) dx$, coefficients $a(x)$, $b(x)$ are fixed measurable functions bounded on Γ_0. Lemma 2[1] that in the space $W_2^1(\Gamma)$ is the set of Ω of functions $u(x) \in C(\Gamma)$ ($C(\Gamma)$ is the space of continuous on Γ functions) satisfying the relations $\sum_{\gamma_j \in R(\xi)} a(1)_{\gamma_j} \frac{du(1)_{\gamma_j}}{dx} = \sum_{\gamma_j \in r(\xi)} a(0)_{\gamma_j} \frac{du(0)_{\gamma_j}}{dx}$ all nodes $\xi \in J(\Gamma)$ (here $R(\xi)$ is the set of edges oriented ``to the node $\xi$ '', $r(\xi)$ is the set of edges oriented ``from the node ξ ''; by $u(\cdot)_\gamma$ is indicated the restriction of $u(\cdot)$ to the edge γ). The closing in norm $W_2^1(\Gamma)$ of the set of functions Ω is denoted by $W_2^1(a,\Gamma)$.

Suppose further that $\Theta(a,\Gamma_T)$ is the set of functions $u(x,t) \in V_2(\Gamma_T)$, whose traces are defined on sections of the field Γ_T plane $t = t_0$ ($t_0 \in [0,T]$) as a function of class $W_2^1(a,\Gamma)$ and satisfy