Applied Soft Computing 12 (2012) 174–181

Contents lists available at SciVerse ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Stability analysis of RBF network-based state-dependent autoregressive model for nonlinear time series

Min Gan [a,b,*], Hui Peng [a]

[a] School of Information Science and Engineering, Central South University, Changsha, Hunan 410083, China
[b] School of Electrical Engineering and Automation, Hefei University of Technology, Hefei, Anhui 230009, China

ARTICLE INFO

Article history:
Received 21 June 2010
Received in revised form 29 March 2011
Accepted 31 August 2011
Available online 7 September 2011

Keywords:
Radial basis function network
State-dependent autoregressive model
Stability analysis
Limit cycle
Canadian lynx data
Time series prediction

ABSTRACT

Varying-coefficient models have attracted great attention in nonlinear time series analysis recently. In this paper, we consider a semi-parametric functional-coefficient autoregressive model, called the radial basis function network-based state-dependent autoregressive (RBF-AR) model. The stability conditions and existing conditions of limit cycle of the RBF-AR model are discussed. An efficient structured parameter estimation method and the modified multi-fold cross-validation criterion are applied to identify the RBF-AR model. Application of the RBF-AR model to the famous Canadian lynx data is presented. The forecasting capability of the RBF-AR model is compared to those of other competing time series models, which shows that the RBF-AR model is as good as or better than other models for the postsample forecasts.

1. Introduction

Varying-coefficient (or functional-coefficient) models, whose parameters may vary with the value of some variables, offer a very flexible structure for modeling nonlinear time series. In recent years, this kind of models has been popularly studied and has many important applications. For example, Cai et al. [1] applied the local linear regression technique for estimation of functional-coefficient regression models for nonlinear time series. Chen and Liu [2] studied nonparametric estimation and hypothesis testing procedures for functional-coefficient autoregressive (FAR) models. Huang and Shen [3] proposed a global smoothing method based on polynomial splines for the estimation of functional-coefficient regression models. Harvill and Ray [4] extended the functional-coefficient autoregressive model to the multivariate nonlinear time series framework. Akesson and Toivonen [5] studied state-dependent parameter representations of stochastic nonlinear sampled-data systems. Zhang [6] studied the proportional functional-coefficient linear regression models. Cai et al. [7] studied functional-coefficient regression models with nonstationary time series data. Cao et al. [8] proposed penalized spline estimation for functional coefficient regression model under dependence. Although the varying-coefficient models have already a vast literature, some researchers believe that the research in this area is just at the beginning [6].

The aforementioned models may be traced back to the state-dependent models of Priestley [9]:

$$x_t = \mu(\mathbf{X}_{t-1}) + \sum_{i=1}^{k}\phi_i(\mathbf{X}_{t-1})x_{t-i} + \varepsilon_t + \sum_{j=1}^{l}\psi_j(\mathbf{X}_{t-1})\varepsilon_{t-j} \quad (1)$$

where $\{x_t,\ \ t = 1, 2, \ldots\}$ are the time series, k and l are positive integers, $\mathbf{X}_{t-1} = (\varepsilon_{t-l}, \ldots, \varepsilon_{t-1}, x_{t-k}, \ldots, x_{t-1})^T$ denotes the “state vector”, $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables, and ε_t is independent of $\{x_{t-i}, i > 0\}$, $\mu(\bullet)$, $\{\phi_i(\bullet)\}$, $\{\psi_j(\bullet)\}$ are measurable functions from $\Re^{k+l} \rightarrow \Re$. Many familiar time series models are special cases of the state-dependent model (1). We just name a few below.

Take $\mu(\bullet)$, and $\{\phi_i(\bullet)\}$, $\{\psi_j(\bullet)\}$ all as constants, then we obtain the linear ARMA model:

$$x_t = \mu + \sum_{i=1}^{k}\phi_i x_{t-i} + \varepsilon_t + \sum_{j=1}^{l}\psi_j\varepsilon_{t-j} \quad (2)$$

Take $\mu(\bullet)$, $\{\phi_i(\bullet)\}$ as constants, and set $\psi_j(\mathbf{X}_{t-1}) = b_j + \sum_{i=1}^{p} c_{ji}x_{t-i}$, then we may obtain the bilinear model [10]:

$$x_t = \sum_{i=1}^{k}\phi_i x_{t-i} + \varepsilon_t + \sum_{j=1}^{l} b_j\varepsilon_{t-j} + \sum_{i=1}^{p}\sum_{j=1}^{m} c_{ij}x_{t-i}\varepsilon_{t-j} \quad (3)$$

* Corresponding author at: School of Electrical Engineering and Automation, Hefei University of Technology, Hefei, Anhui 230009, China. Tel.: +86 15215698383.
E-mail address: aganmin@yahoo.com.cn (M. Gan).

1568-4946/$ – see front matter
doi:10.1016/j.asoc.2011.08.055

The threshold autoregressive (TAR) model introduced by Tong [11], which is one of the simplest but widely used nonlinear time series model, can also be regarded a special case of state-dependent model (1):

$$x_t = \sum_{i=1}^{k} \{\mu^{(i)} + \phi_1^{(i)} x_{t-1} + \cdots + \phi_p^{(i)} x_{t-p} + \varepsilon_t^{(i)}\} I(x_{t-d} \in \Omega_i) \tag{4}$$

where $I(\bullet)$ is the usual indicator function, and $\{\Omega_i\}$ form a non-overlapping partition of the real line.

Ozaki [12] and Haggan and Ozaki [13] proposed the exponential autoregressive (ExpAR) model which exhibit certain well-known features of non-linear vibrations theory, such as amplitude-dependent frequency, jump phenomena, and limit cycle behavior:

$$\begin{aligned} x_t &= (\phi_1 + \pi_1 \exp(-\gamma x_{t-1}^2)) x_{t-1} + \cdots + (\phi_p + \pi_p \\ &\quad \times \exp(-\gamma x_{t-1}^2)) x_{t-p} + \varepsilon_t \end{aligned} \tag{5}$$

where $\{\phi_1, \ldots, \phi_p\}$, $\{\pi_1, \ldots, \pi_p\}$ and $\gamma > 0$ are constants.

Chen and Tsay [14] proposed the functional-coefficient autoregressive (FAR) model:

$$x_t = \phi_1(\mathbf{X}_{t-1}^*) x_{t-1} + \cdots + \phi_p(\mathbf{X}_{t-1}^*) x_{t-p} + \varepsilon_t \tag{6}$$

where $\mathbf{X}_{t-1}^* = (x_{t-i_1}, \ldots, x_{t-i_k})^{\mathrm{T}}$ is the threshold vector (or state vector) with $i_1, \ldots, i_k$ as the threshold lags, $\phi_i(\mathbf{X}_{t-1}^*)$ are measurable functions from $\Re^k \to \Re$. In practice, the k is usually a small number (e.g., $k = 1$ in [1,14]), because the models with large k are often not practically useful due to "curse of dimensionality" [1]. Obviously, the FAR model (6) is a special case of the state-dependent model (1) without any white noise term ε_{t-i} in the threshold vector and only with the simple AR structure, which makes the empirical estimation is much easier than the general state-dependent model.

The main difficulty in using the FAR model (6) is specifying the functional coefficients $\phi_i(\mathbf{X}_{t-1}^*)$. An efficient estimation approach is using the nonparametric regression techniques [1–3,8]. An alternative to the nonparametric estimation method, if we treat the functional specification as a problem of function approximation from a multi-dimensional input space $\mathbf{X}_{t-1}^*$ to a one-dimensional scalar space, is the neural network approximation. Neural networks are very popular tools for time series modeling and forecasting [15–22]. Because of the "universal approximation" capability of the radial basis function (RBF) networks, Vesin [23] and Shi et al. [24] used a set of RBF networks to approximate the functional coefficients of the state-dependent autoregressive (or FAR) model. The derived model, call the RBF network-based autoregressive (RBF-AR) model, takes the form

$$\begin{cases} x_t = \phi_0(\mathbf{X}_{t-1}^*) + \sum_{i=1}^{p} \phi_i(\mathbf{X}_{t-1}^*) x_{t-i} + \varepsilon_t \\ \phi_0(\mathbf{X}_{t-1}^*) = c_0 + \sum_{k=1}^{m} c_k \exp\{-\lambda_k \|\mathbf{X}_{t-1}^* - \mathbf{Z}_k\|^2\} \\ \phi_i(\mathbf{X}_{t-1}^*) = c_{i,0} + \sum_{k=1}^{m} c_{i,k} \exp\{-\lambda_k \|\mathbf{X}_{t-1}^* - \mathbf{Z}_k\|^2\} \\ \mathbf{X}_{t-1}^* = (x_{t-1}, \ldots, x_{t-d})^{\mathrm{T}} \\ \mathbf{Z}_k = (z_{k,1}, \ldots, z_{k,d})^{\mathrm{T}} \end{cases} \tag{7}$$

where p is the model order, m is the number of hidden nodes of the RBF networks, and $d \le p$ is the dimension of the state vector $\mathbf{X}_{t-1}^*$; $\phi_0(\mathbf{X}_{t-1}^*)$ and $\phi_i(\mathbf{X}_{t-1}^*)$ are the state-dependent functional coefficients which are all composed of RBF networks; $\mathbf{Z}_k (k = 1, 2, \ldots, m)$ are the centers of the RBF networks; $\lambda_k > 0$ $(k = 1, 2, .., m)$ are the real scaling parameters; $c_k (k = 0, 1, 2, \ldots, m)$ and $c_{i,k} (i = 1, 2, \ldots, p; k = 0, 1, 2, \ldots, m)$ are real constants; $\|\bullet\|$ denotes the vector 2-norm. The RBF-AR model (7) treats the nonlinear process by splitting the state space up into a large number of small segments, and regarding the process as "locally linear" within each segment. It allows the coefficients to change gradually, rather than abruptly as in the TAR model. This may be appealing in many applications.

From Eq. (7), we can see that the RBF-AR model can be regarded as a generalized version of the classic ExpAR model (5). The model turns out, however, to has a much stronger capability in prediction and simulation than an ExpAR model [25]. If we take $\mathbf{X}_{t-1}^* = (x_{t-1}, \Delta x_{t-1}, \ldots, \Delta^{d-1} x_{t-1})^{\mathrm{T}}$ in model (7), it is easily can be seen that the instantaneous dynamics of the RBF-AR model depends not only on the present amplitude of the series but also on its velocity Δx_t and/or acceleration $\Delta^2 x_t$. Therefore, it could produce, for example, asymmetric nonlinear wave patterns in time series, since the model dynamics may be different when the series is increasing or decreasing [25]. On the other hand, the RBF-AR model actually shares the same flexibility in characterizing complex dynamics with the RBF network, since it contains the RBF network as a component. The overwhelming advantage of this class of pseudo-linear AR models over the conventional nonlinear models may be more clearly seen in the application of the models in modern predictive control problems. Peng et al. [26] extended the RBF-AR model to the case where there are several exogenous variables (RBF-ARX model) to the system, and designed RBF-ARX model-based predictive control (MPC) strategies to the nitrogen oxide (NO_x) decomposition process in thermal power plants [27–31]. A major feature of the RBF-AR(X) model approach is that, in contrast to most other time-varying liner models, its parameters may be estimated off-line.

In this paper, we study some probabilistic properties, the identification procedure and application of the RBF-AR model. In Section 2, stability conditions and existing conditions of limit cycle of the RBF-AR model are discussed. Identification of the RBF-AR model includes the choice of the orders, estimation of all the parameters, and selection of the appropriate state vector. Parameter optimization of the RBF-AR model is essentially a nonlinear optimization problem, which is a very difficult task. However, the parameters of RBF-AR model can be classified into linear and nonlinear sets, and the number of linear parameters is usually much larger than the number of nonlinear parameters. Peng et al. [26] proposed a very efficient estimation algorithm so called structured nonlinear parameter optimization method (SNPOM) for this kind of nonlinear model. The orders of the RBF-AR model are determined by the modified multi-fold cross-validation criterion which is proposed by Cai et al. [1]. We also consider the selection of the appropriate state vector for RBF-AR models, which is never considered in previous research. The identification of the RBF-AR model based on SNPOM and the modified multi-fold cross-validation criterion is presented in Section 3. To compare the forecasting capabilities of the RBF-AR with some other competing time series models, the RBF-AR model is applied to the famous Canadian lynx data which is also used in Cai et al. [1], Zhang [32], Katijani [33], and Aladag et al. [34]. It is shown that the RBF-AR model is as good as or better than other models for the post-sample forecasts.

2. Stability analysis and limit cycle of the RBF-AR model

In this section, we give the stability conditions and existing conditions of limit cycle of the RBF-AR model. It is not easy to check whether a time series generated by a nonlinear model is strict stationary. For a nonlinear time series model described by a stochastic difference equation, we usually first represent the time series as a vector-valued Markov chain. Then, we derive the stationarity of the model by proving the corresponding Markov chain is ergodic.