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A new real-coded quantum-inspired evolutionary algorithm for continuous optimization

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ABSTRACT

This paper presents a recursive deepening hybrid strategy to solve real-parameter optimization problems. It couples a local search technique with a quantum-inspired evolutionary algorithm. In order to adapt the quantum-inspired evolutionary algorithm for continuous optimization without losing the states superposition property, a suitable sampling of the search space that tightens recursively and an integration of a uniformly generated random part after measurement have been utilized. The use of local search provides, for each search window, a good exploitation of the quantum inspired generated solution's neighbourhood. The proposed approach has been tested through the reference black-box optimization benchmarking framework. The comparison of the obtained results with those of some state-of-the-art algorithms has shown its actual effectiveness.

1. Introduction

Optimization methods are very often needed to deal with real-life problems in different fields. Exact methods give the optimal solution but they are impracticable for a large class of optimization problems because of their exponential or factorial algorithmic complexity.

Approximate methods, usually based on metaheuristics, have then been developed to produce acceptable solutions for those problems. Amongst these methods, we find the Quantum-Inspired Evolutionary Algorithms (QEA) which take their origin from quantum physics. They are based on the use of the quantum bit (*qubit*); the elementary unit that allows manipulating a superposition of the basic states $|0\rangle$ and $|1\rangle$, rather than a single binary value as acquainted. It is from this superposition of states combined to adequate operators that comes the aptitude of these algorithms to better explore the search space since it is possible to represent and manipulate simultaneously 2^n different values or candidate solutions with only n qubits. So, a single individual can potentially represent all the possible values within the search space at the same time.

QEA have been successfully used to solve many binary-coded problems such as the knapsack problem [1], the travelling salesman problem [2], the N-Queens problem [3], multiple sequence alignment [4], the job shop scheduling problem [5], face detection [6], resource allocation in grid computing [7], and multiobjective image segmentation [8]. However, using QEA to deal with continuous optimisation is a bit problematic, since there is no obvious way to represent a superposition of real values. This justifies the limited number of well-positioned works that adopt the quantum-inspired evolutionary strategies for continuous problems. In fact, no previous real-coded QEA has been successfully confronted to the dominating state-of-the-art metaheuristic methods.

In [9,10], where the authors have dealt with image registration using variants of the QEA, the real parameters of the geometric transformation that aligns two images have been calculated from the binary values resulting from the quantum chromosomes measurement. But it should be mentioned that those parameters do not need to be too precise because of the limited resolution of the processed digital images. Therefore, there is no actual obstacle to get from the binary to the real domain.

In [11–15], attempts have been made to adapt QEA to real-parameters problems. Despite the acceptable obtained optimization results, it should be stated that those proposals tend to neglect a very important quantum-inspired feature; it is the superposition property that allows encoding simultaneously all the possible solutions and then guarantees an intrinsic diversity. In fact, those proposals focus more on the evolutionary aspect through the

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quantum rotation gate, but the real solution is extracted almost deterministically from the qubits' α and β values.

It could be easily supposed that considering the states superposition property would make QEA more efficient since it will offer more diversity and so more exploration capabilities to compete with the state-of-the-art algorithms.

The question that will be tackled in the present work is then: could we make QEA deal with continuous optimisation problems without neglecting the states superposition property? i.e. the extraction of solutions from the population individuals has to be probabilistic and not deterministic.

In this paper, we propose a hybrid method based on QEA adapted to continuous optimization problems. It uses a sampling mechanism that allows getting real values from binary probabilistically measured numbers. In order to make the algorithm converge progressively toward the optimum and because of the density of the real domain, we use a deepening strategy that permits increasing precision throughout the optimization process.

QEA offers a good exploration of the search space because of the diversity guaranteed by the states superposition. Of course, when considering that superposition, the probabilities to get the different solutions after measurement may significantly vary, but all of them remain potentially observable. Besides, some local search techniques, though not very robust for global optimization, are very efficient in finding the best solution within a limited neighbourhood. To improve the exploitation property of the proposed approach, a local search technique has been incorporated to get efficiently the best solution around a QEA's generated solution at a given depth.

The paper is organized as follows. After the present introduction, Section 2 gives an overview of the related works. Section 3 describes the proposed algorithm. Section 4 gives the experiment configuration and a description of the adopted benchmarks. A comparative study between the proposed Real-Coded Quantum-Inspired Evolutionary Algorithm (RQEA) and several state-of-the-art metaheuristics is presented in Section 5. Finally, the main conclusions and some possible further developments are given in Section 6.

2. Related works

Quantum computing has its origin in the work of Peter Shor who proposed, in 1994, a quantum algorithm for factoring numbers that needs only a polynomial time. This is fast compared to the best-known classical factoring algorithm that has a complexity of $\mathcal{O}(2^{n^{1/3}\log(n)^{2/3}})$ [16]. In parallel, Grover [17] devised a quantum algorithm for searching an item in a database of n elements in only $\mathcal{O}(\sqrt{n})$ steps, which is faster than its classical counterpart that requires $\mathcal{O}(n)$ steps.

Theoretically speaking, quantum algorithms are much faster to their counterparts, thanks to the notion of state superposition that permits acting on many quantum states in parallel. However, the implementation of such algorithms has remained very limited in reality, since quantum computers have remained in an elementary state for the last decades. This has not prevented the research community of quantum informatics from exploiting the power of the notions of quantum physics. Thus, a new question of research has emerged: "why not to exploit quantum physics concepts as source of inspiration to devise powerful algorithms that could be run on classical computers?" Consequently, tens of quantum-inspired algorithms have been proposed in the last decades to efficiently solve many real-world and academic problems.

One sub-area of research in the field of quantum-inspired computing is that of *quantum-inspired evolutionary* computation. The latter hybridises classical evolutionary algorithms with the concepts of quantum computing, in order to overcome the problem of dimensionality in optimization problems.

Before going further in reviewing the literature of quantum-inspired evolutionary algorithms, it is worth to mention that most of the works in the literature use the name *Quantum-Inspired Genetic Algorithm* (QIGA), but in reality only a few of them are truly. In fact, they use either only mutation, or crossover, or no genetic operators at all. So, they are more evolutionary algorithms than QIGAs [18].

The first quantum-inspired genetic algorithm was proposed by Narayanan and Moore [19] in 1994. It has been used to address the knapsack problem in both sequential and a parallel ways [20–22]. It was also used to tackle the calculation of the partition function [23]. Later on, the authors of [24,25] proposed a quantum-inspired genetic algorithm for solving the blind source separation problem. A parallel version of the QIGA was proposed in [26] for infinite impulse response digital filter design. This work was extended in [27] through using chaos-updated rotated gates.

A quantum-inspired genetic algorithm was used in [28] for controlling the design process. The travelling salesman problem was targeted using a quantum-inspired genetic algorithm in [2]. The image registration task was dealt with using a QIGA in [29].

In addition to these founding works, many other successful applications of quantum-inspired evolutionary algorithms have proven their efficiency, particularly in the fields of medicine, bioinformatics, image processing, industrial problems, networking, communication, transportation, robotics, information systems and data processing. We cite here some of those works.

The QIGA has been applied for solving optimization problem in the field of networking and communication problems [30,31]. Other academic combinatorial problems such as the Knapsack and Max Sat tasks have been tackled in [32] using the QIGA. To solve electromagnetic and wave propagation problems the QIGA was exploited in [33]. The authors in [34] have used at their turn the QIGA for grey level image thresholding.

A QIGA was also used in [35] to optimize the agricultural products supply chain network, In [36] the QIGA was used for the PID control in industrial manufacturing. Finally, a QIGA was used to solve some other industrial problems in [37].

As far as continuous optimization problems are concerned, some works have been proposed to adapt the QIGA, basically proposed to treat discrete problems, for dealing with continuous search space. We cite some of these algorithms.

In [11,13–15], attempts have been made to adapt QEA to real-parameters problems. As mentioned above, those proposals neglect the state superposition property and focus more on the evolutionary aspect. The following paragraphs briefly present some other variants of the real-coded QEA.

In [38,39], the authors have proven that real encoding is better than binary encoding for adapting the QEA to continuous optimization problems. To accelerate the convergence rate, Liu et al. [40] replaced the binary representation of qubits and the binary-based Q-gate by new real ones devised especially for continuous problems. Babu et al. [41] employed neighbourhood operators in solution strings with real parameters.

In [42–45], the authors used a new diagonal recombination operator called global-interference crossover, mainly proposed in [46], to deal with some real-coded problems in the domain of finance. The genes of chromosomes were expressed by phase angles instead of real values in [47], where the authors claim they found better results. To overcome the problem of premature convergence while dealing with multi-modal functions, Ni and Wang [48] proposed using the niche elimination mechanism to maintain diversity. Adaptivity, in addition to other new operators, have been used in the QEA proposed in [49].